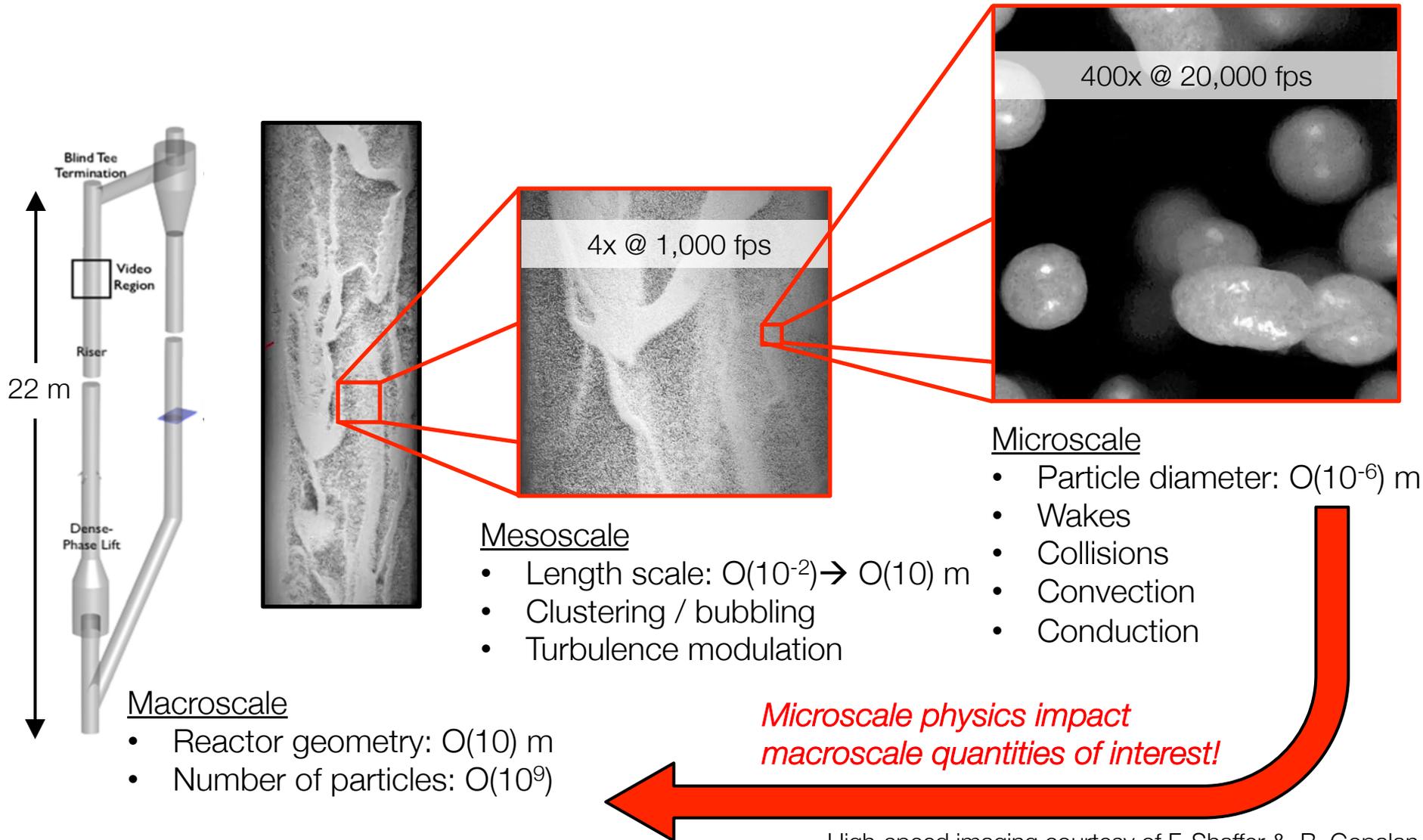


# Understanding and modeling the effect of clusters on heat transfer in gas-solid suspensions

Jesse Capecelatro  
University of Michigan, Ann Arbor

# Two-phase interactions impact flows across scales

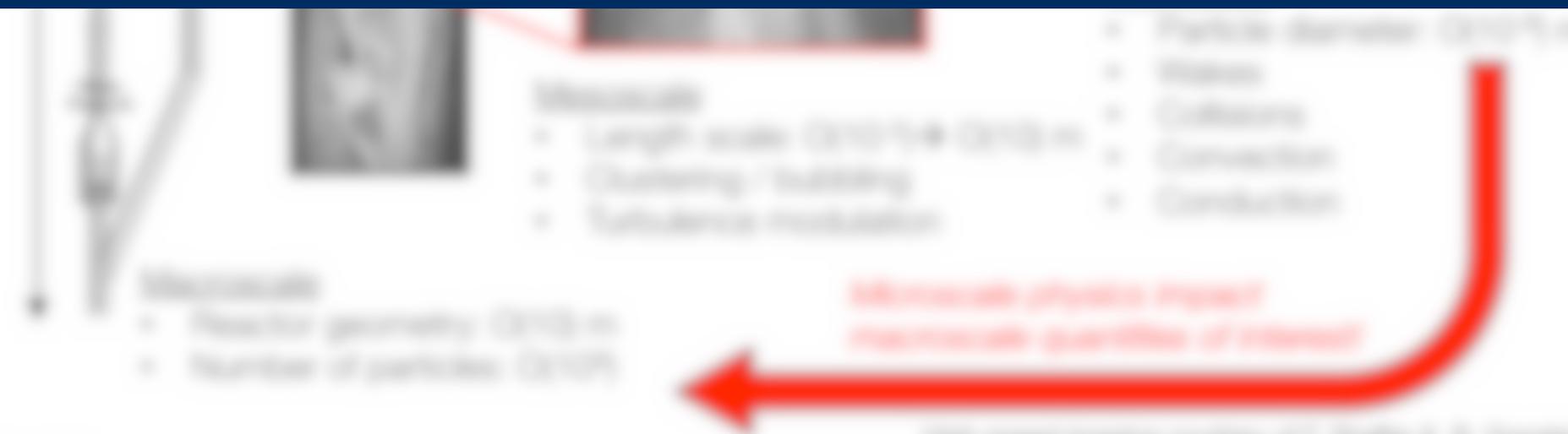


High-speed imaging courtesy of F. Shaffer & B. Gopalan  
 D.O.E. National Energy Technology Lab

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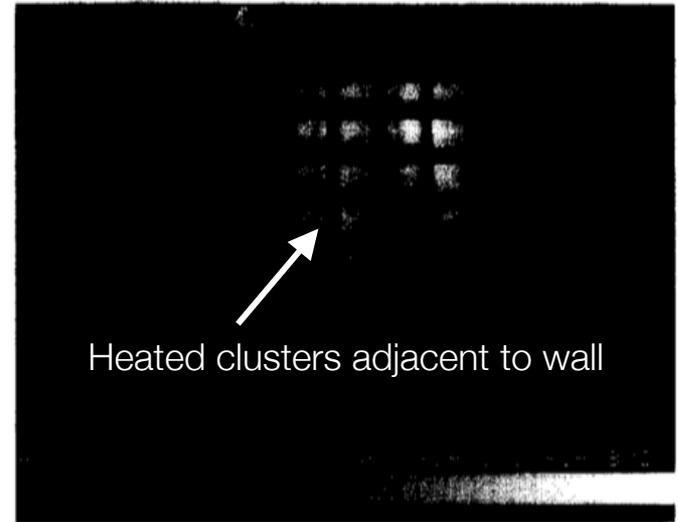


1. To what extent can clustering augment/obstruct heat transfer?
2. Can this be described statistically?



## Early experimental work reveal order-one effect

- Early fluidized bed experiments in the 1990s by Louge et al.<sup>1</sup> and Ebert et al.<sup>2</sup> demonstrated an order of magnitude increase in the bed-to-wall heat transfer in comparison to single-phase turbulence, with greater effect at high particle concentrations.
- Experiments of circulating fluidized bed reactors by Noymer & Glicksman<sup>3</sup> later revealed that the presence of particles are capable of reducing heat transfer to the wall at low volume fractions. They found that neglecting cluster acceleration can result in over predictions in local heat transfer rates by as much as 50%.



Infrared image of heated clusters in a fluidized bed<sup>3</sup>

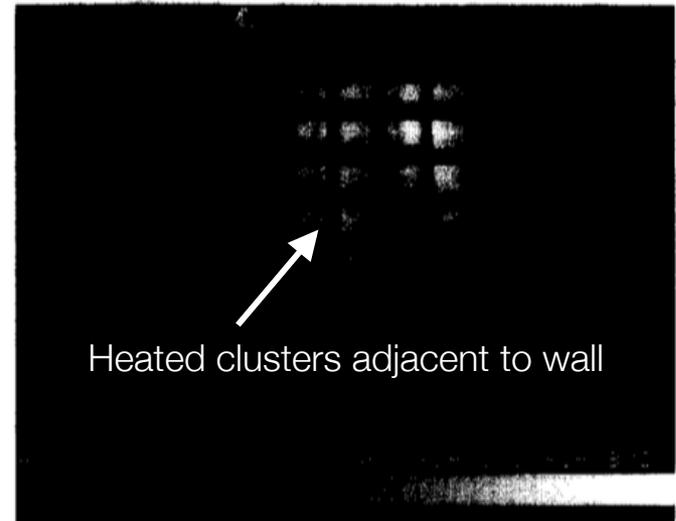
<sup>1</sup>Louge, Yusof, Jenkins, *Int. J. Heat Mass Transfer* (1993)

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Heated clusters adjacent to wall

Infrared image of heated clusters in a fluidized bed<sup>3</sup>

### *Need for numerical simulations:*

- Experimental studies like these are limited.
- Reduced optical access by particles and multi-scale nature make it challenging to identify physical mechanisms responsible for these observations.

<sup>1</sup>Louge, Yusof, Jenkins, *Int. J. Heat Mass Transfer* (1993)

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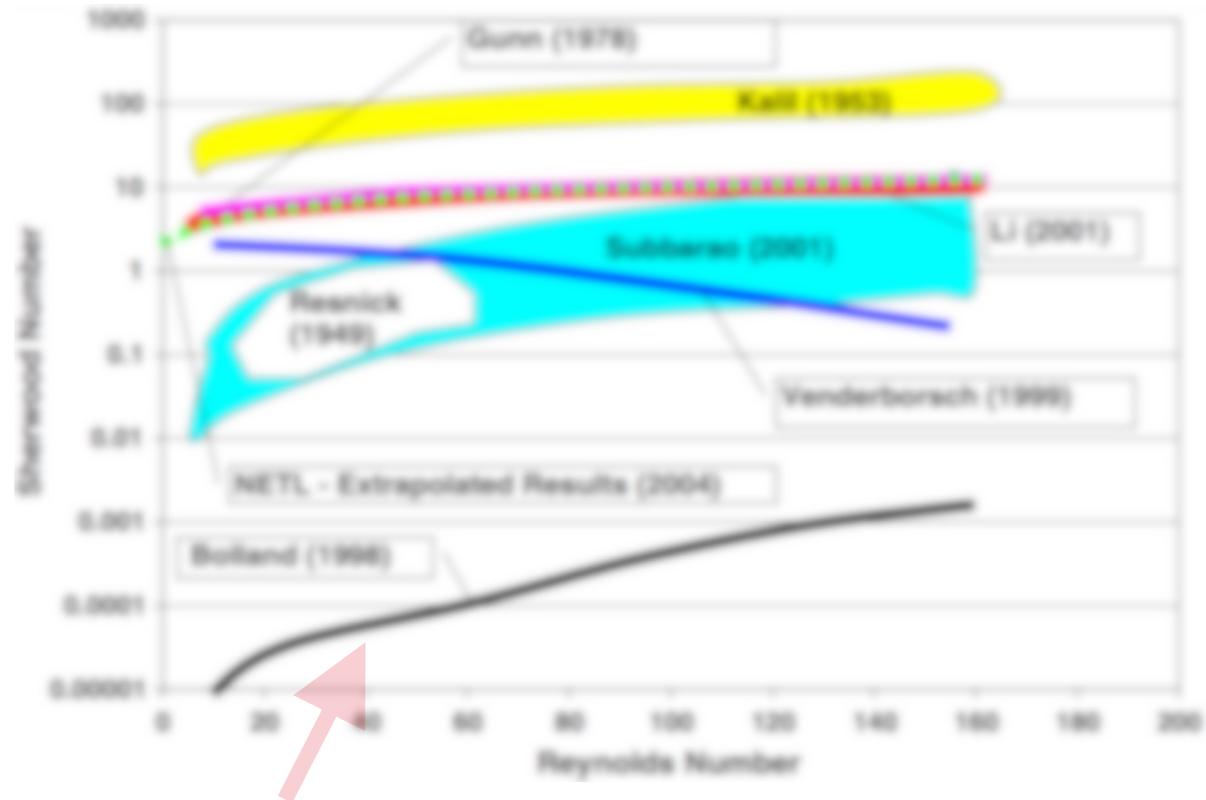
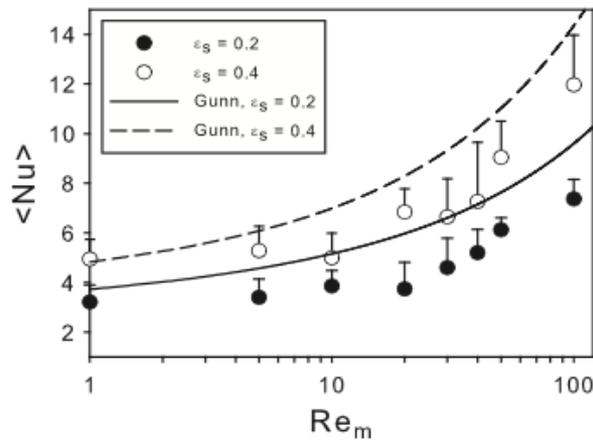
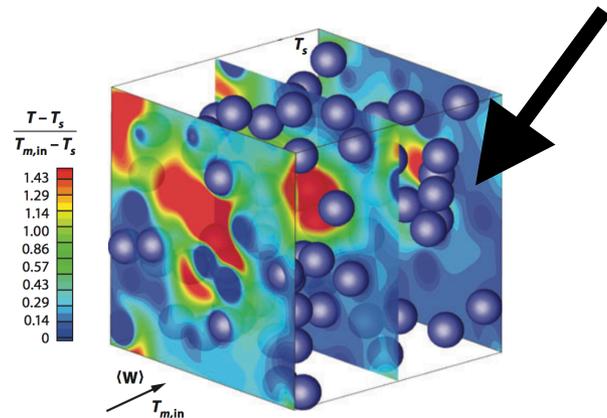
“All models are wrong

but some are useful”

- George Box

# Heat transfer correlations are not universal

Particle-resolved DNS by Subramaniam & coworkers<sup>1</sup> demonstrates excellent agreement with Gunn's correlation (almost 40 years later!)



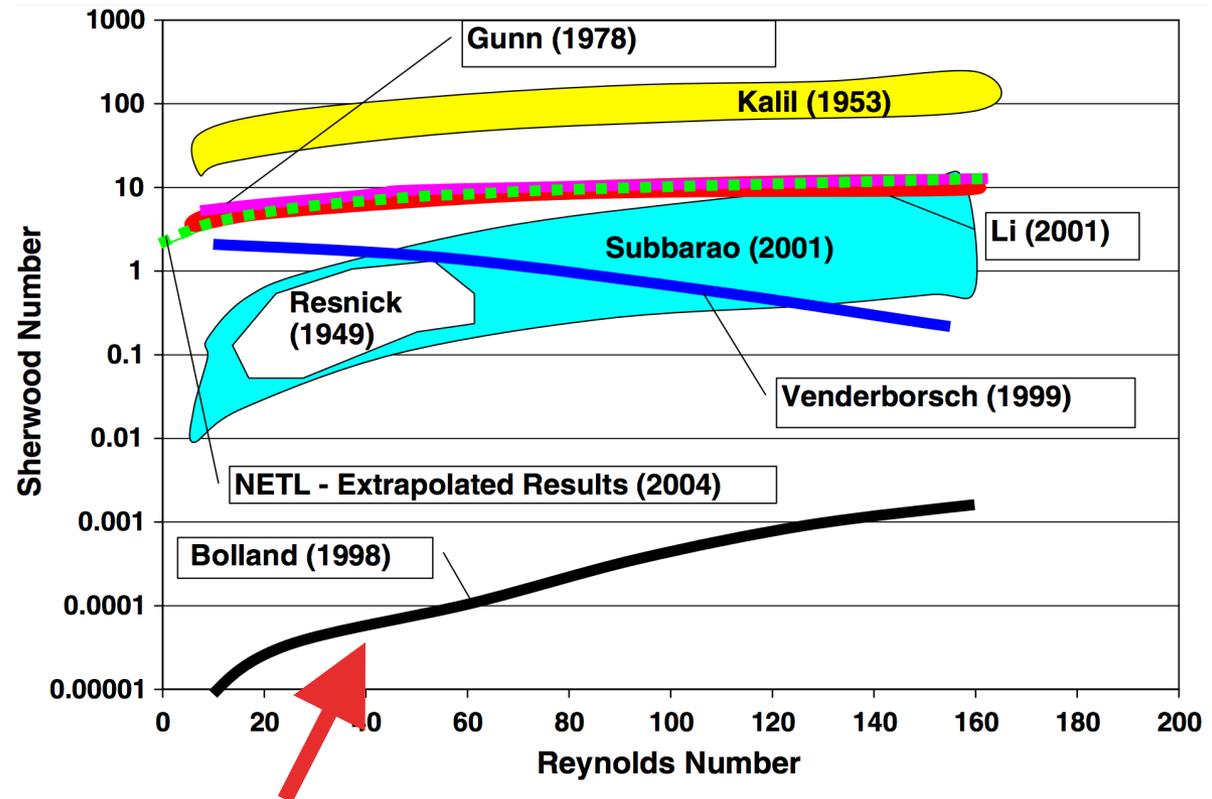
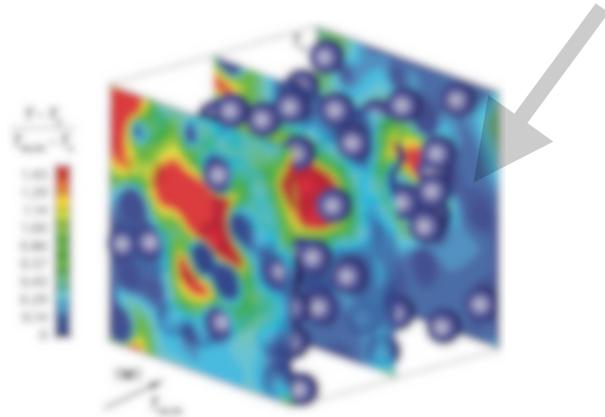
Orders of magnitude discrepancy in local heat/mass transfer coefficients<sup>2</sup> due to varying flow regimes!

<sup>1</sup>Tenneti, Sun, Garg, Subramaniam, *Int. J. Heat Mass Transfer* (2013)

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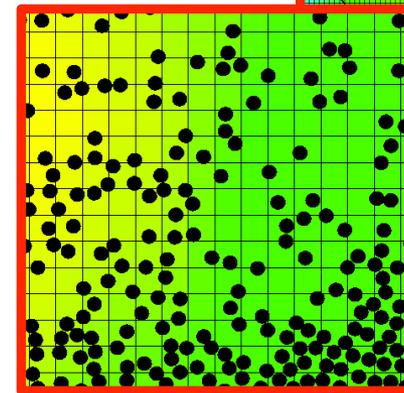
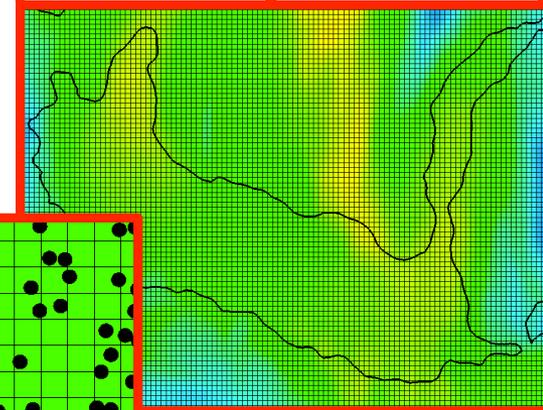
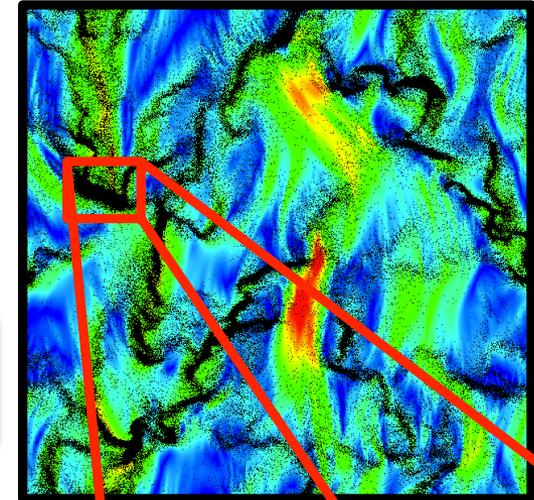
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# Eulerian-Lagrangian framework

- NGA<sup>1</sup>
  - Volume-filtered DNS/LES code
  - Massively parallel
  - Conservation of mass, momentum, and kinetic energy

$$\frac{\partial}{\partial t}(\varepsilon_f \rho_f \mathbf{u}_f) + \nabla \cdot (\varepsilon_f \rho_f \mathbf{u}_f \mathbf{u}_f) = \nabla \cdot \boldsymbol{\tau} + \varepsilon_f \rho_f \mathbf{g} + \mathcal{F}_{\text{inter}} + \mathbf{F}_{\text{mfr}}$$



- Lagrangian particle tracking<sup>2</sup>

- Newton's 2<sup>nd</sup> law

$$m_p \frac{d\mathbf{u}_p^{(i)}}{dt} = \mathbf{F}_{\text{inter}}^{(i)} + \mathbf{F}_{\text{col}}^{(i)} + m_p \mathbf{g}$$

- Fluid quantities interpolated to particle location
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Stress tensor contains model for effective viscosity (Gibilaro, 2007)

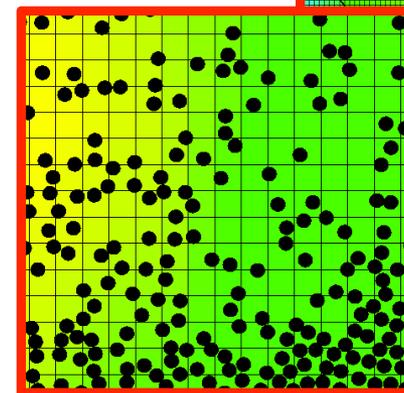
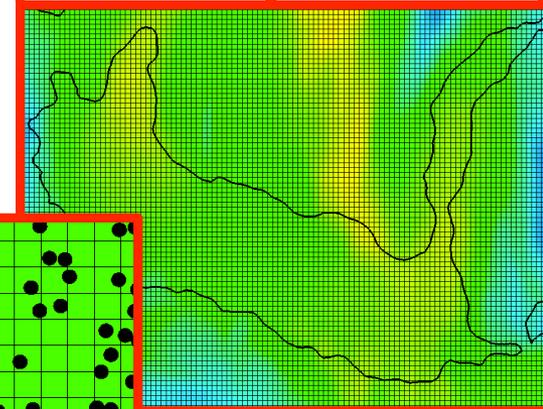
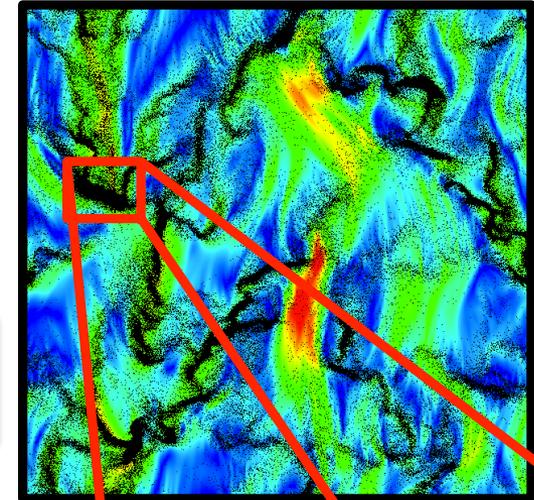
$$\boldsymbol{\tau} = -p\mathbb{I} + \underline{\mu^*} \left[ \nabla \mathbf{u}_f + \nabla \mathbf{u}_f^T - \frac{2}{3}(\nabla \cdot \mathbf{u}_f)\mathbb{I} \right] \quad \underline{\mu^* = \mu \varepsilon_f^{-2.8}}$$

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Momentum exchange

$$\mathbf{F}_{\text{inter}}^{(i)} = V_p \nabla \cdot \boldsymbol{\tau}[\mathbf{x}_p^{(i)}] + \mathbf{f}_{\text{drag}}^{(i)}$$

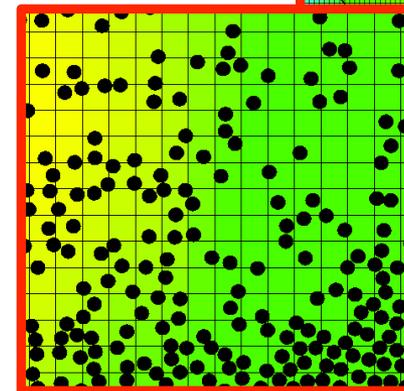
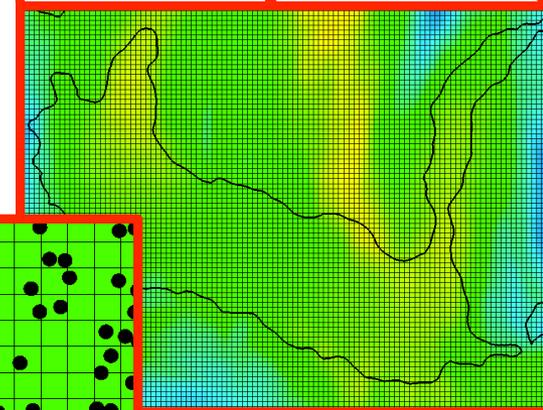
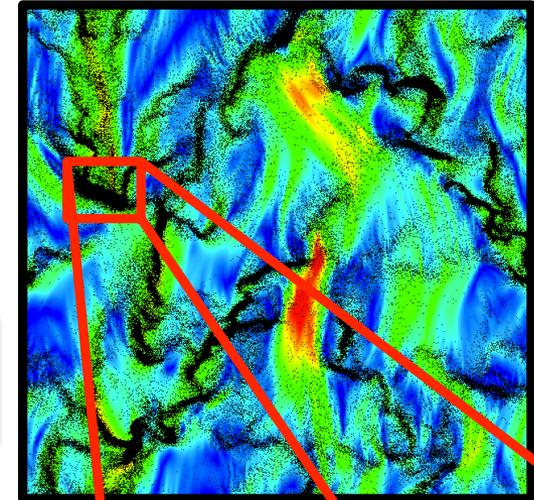
Resolved stresses + Unresolved stresses

- Lagrangian particle tracking<sup>2</sup>

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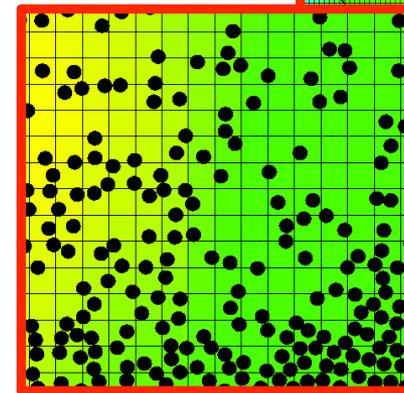
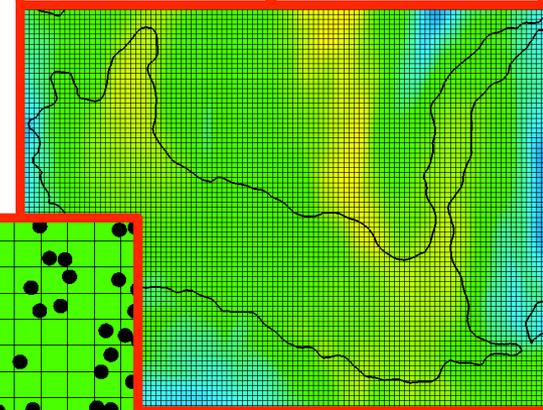
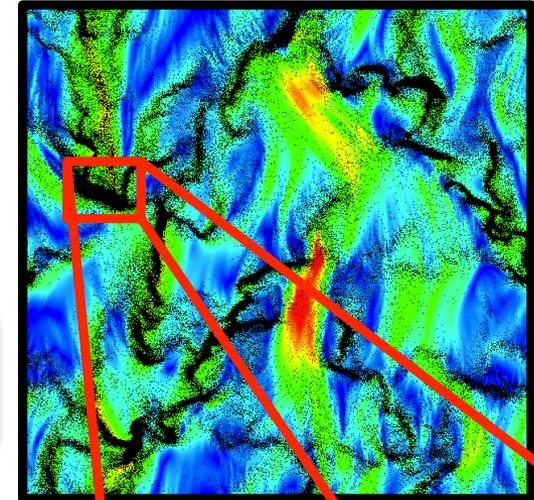
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Forcing to ensure statistical stationarity



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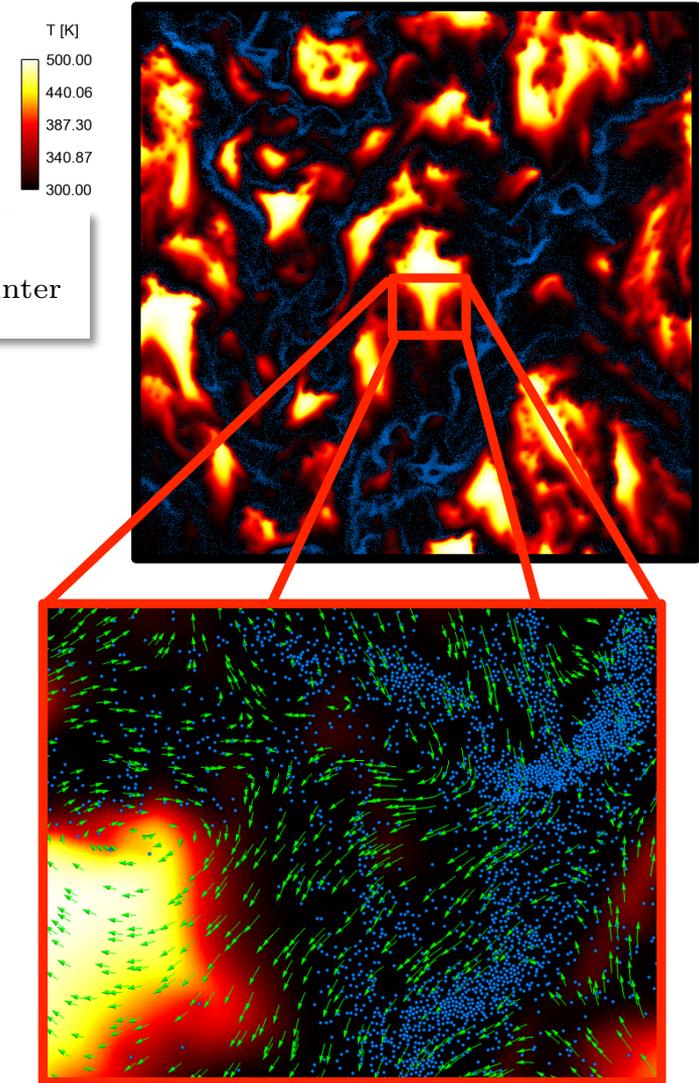
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$$\frac{\partial}{\partial t}(\varepsilon_f \rho_f C_{p,f} T_f) + \nabla \cdot (\varepsilon_f \rho_f C_{p,f} \mathbf{u}_f T_f) = \nabla \cdot (k_f^* \nabla T_f) + Q_{\text{inter}}$$

- Particle temperature

$$m_p C_{p,p} \frac{dT_p^{(i)}}{dt} = q_{\text{heat}}^{(i)}$$

$$\frac{q_{\text{heat}}^{(i)}}{V_p} = \nabla \cdot (k_f^* \nabla T_f) + \frac{6k_f \text{Nu}}{d_p^2} (T_f[x_p^{(i)}] - T_p^{(i)})$$



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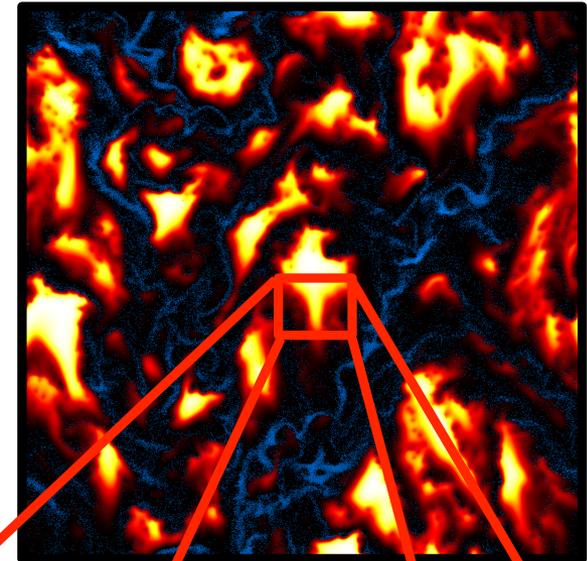
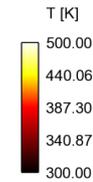
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Heat exchange

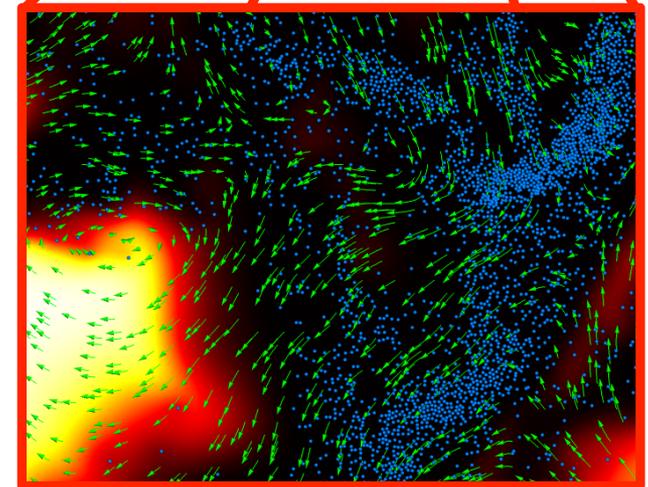


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Resolved heat flux + Unresolved heat flux



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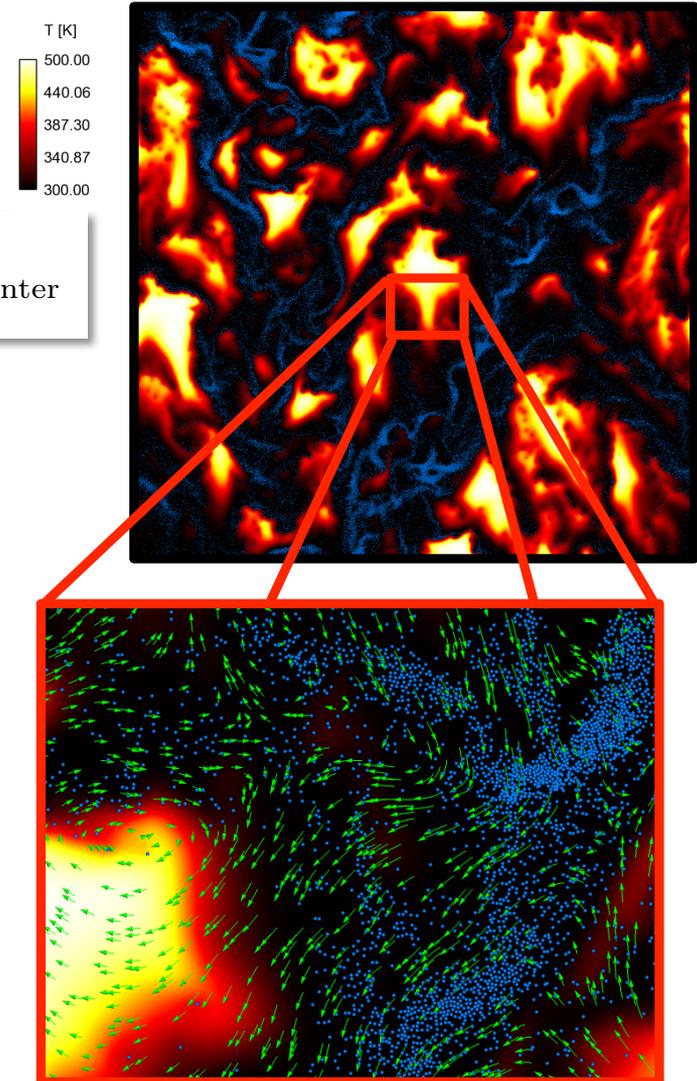
Effective conductivity (Sutkar et al., 2016)

$$k_f^* = \frac{1 - \sqrt{1 - \varepsilon_f}}{\varepsilon_f} k_f.$$

- Particle temperature

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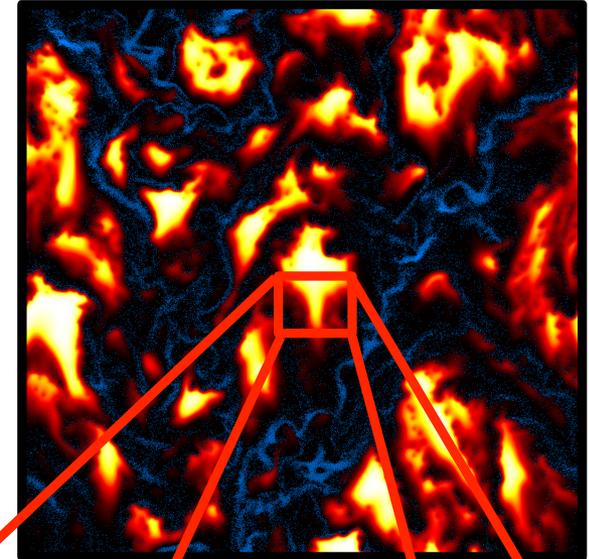
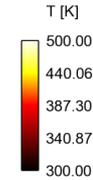
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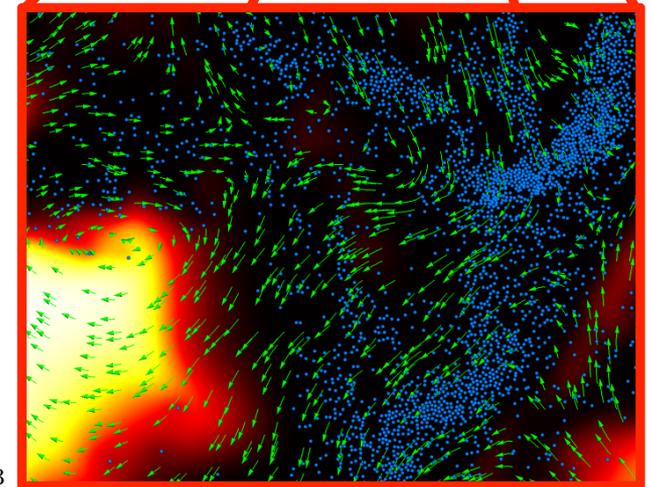
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Nusselt number correlation of Gunn (1978):

$$\text{Nu} = (7 - 10\varepsilon_f + 5\varepsilon_f^2) [1 + 0.7\text{Re}_p^{0.2}\text{Pr}^{0.33}] + (1.33 - 2.4\varepsilon_f + 1.2\varepsilon_f^2) \text{Re}_p^{0.7}\text{Pr}^{0.33}$$



<sup>1</sup>Guo, Capecehatro, *Int. J. Heat Mass Transfer* (Under review)

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# Interphase exchange

Interphase exchange terms are projected to the mesh via:

- Volume fraction:

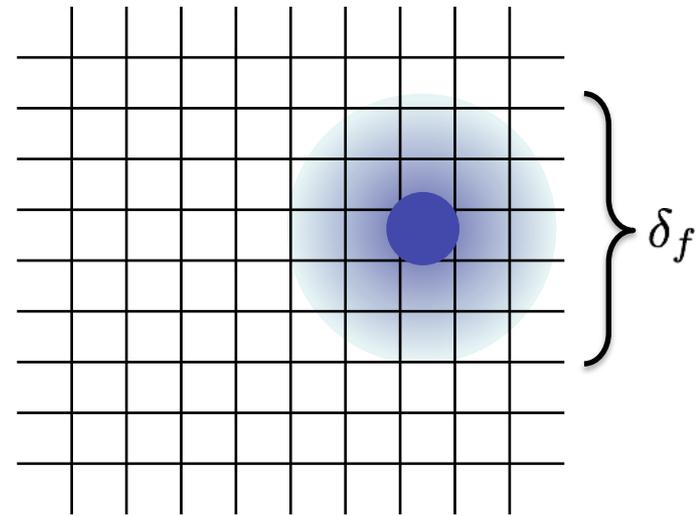
$$\varepsilon_f = 1 - \sum_{i=1}^{N_p} \mathcal{G}(|\mathbf{x} - \mathbf{x}_p^{(i)}|) V_p$$

- Momentum exchange:

$$\mathcal{F}_{\text{inter}} = - \sum_{i=1}^{N_p} \mathcal{G}(|\mathbf{x} - \mathbf{x}_p^{(i)}|) \mathbf{f}_{\text{drag}}^{(i)}$$

- Heat exchange:

$$Q_{\text{inter}} = - \sum_{i=1}^{N_p} \mathcal{G}(|\mathbf{x} - \mathbf{x}_p^{(i)}|) q_{\text{heat}}^{(i)}$$



## Two Step Filtering approach<sup>1</sup>

1. Data is sent to neighboring grid points
2. Solution diffused with characteristic size  $\delta_f$

# Novel Interphase

Interphase exchange terms are projected to the mesh via:

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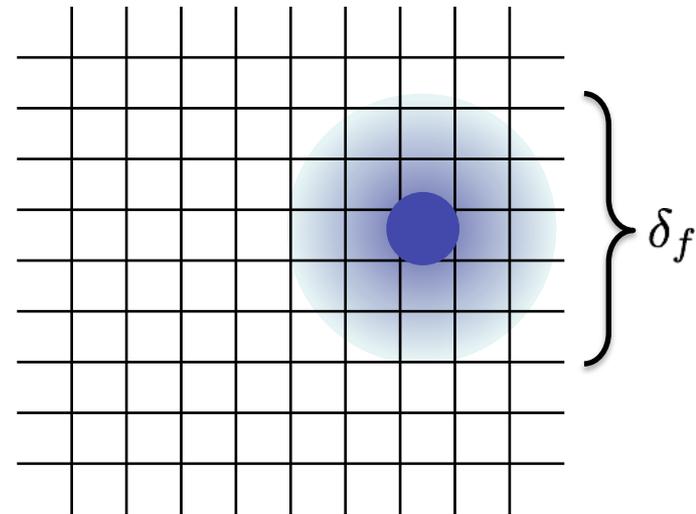
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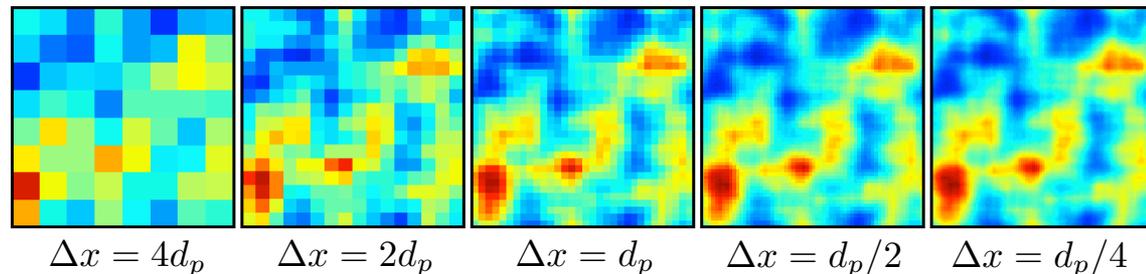
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## Two Step Filtering approach<sup>1</sup>

1. Data is sent to neighboring grid points
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Filter width  $\delta_f$  is independent of the mesh size<sup>1</sup>

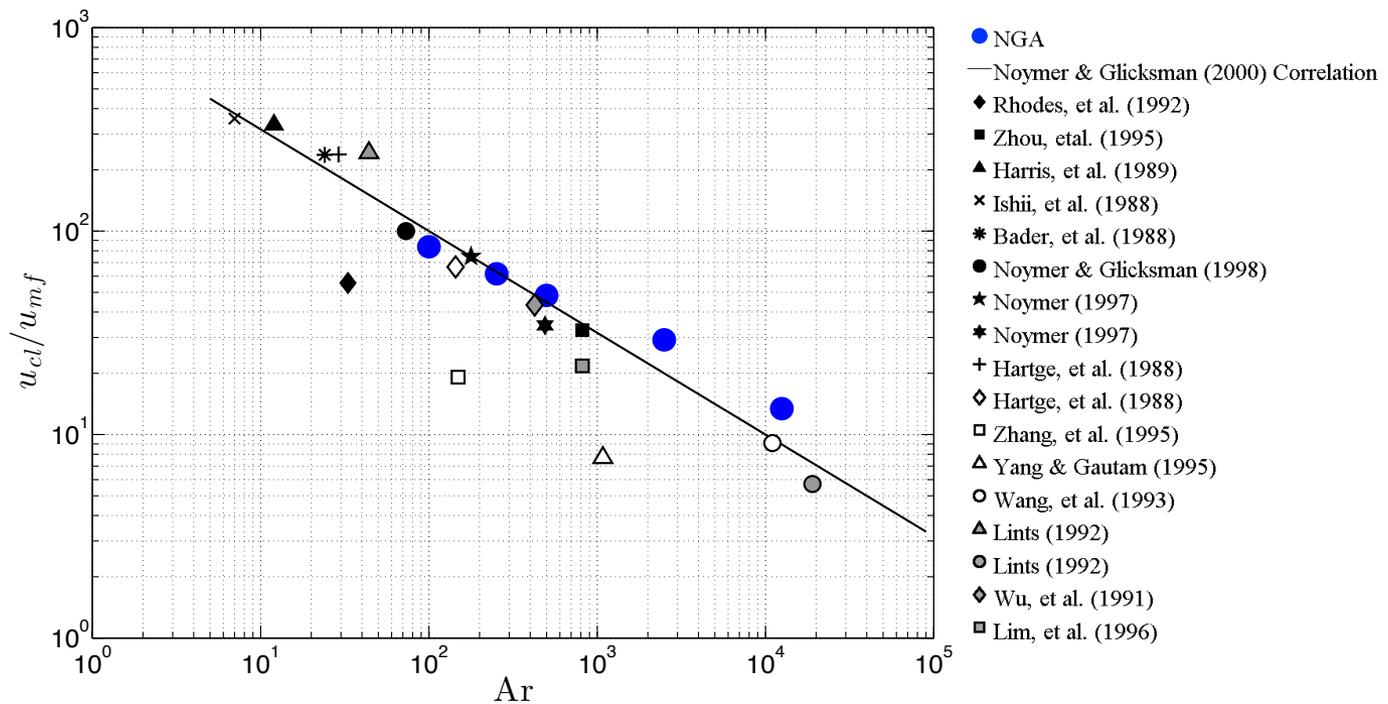
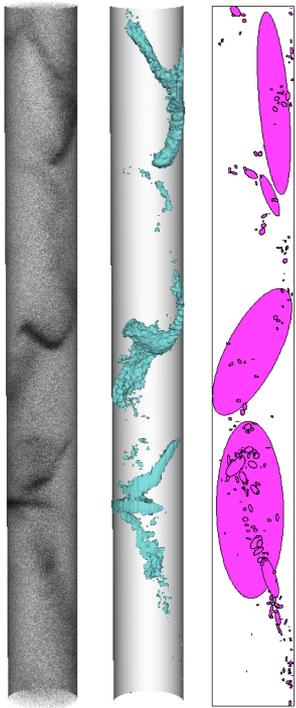


<sup>1</sup>Capececelatro, Desjardins, *J. Comp. Phys.*, (2013)

# Numerical prediction of cluster velocity

Cluster fall velocity (cold flow)<sup>1</sup>

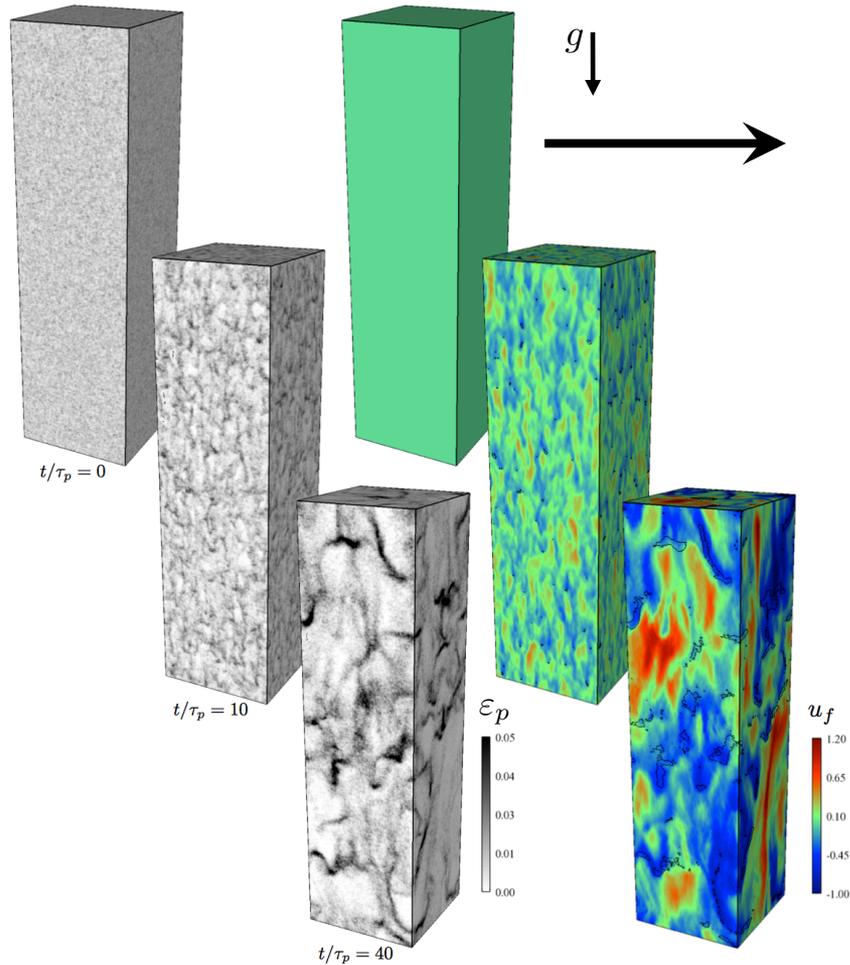
- Structure tracking algorithm used to identify individual clusters
- Clusters fall at constant velocities consistent with experimental measurements



Experimental data compiled by: P.D. Noymer and L.R. Glicksman (2000), *Chem. Eng. Sci.*

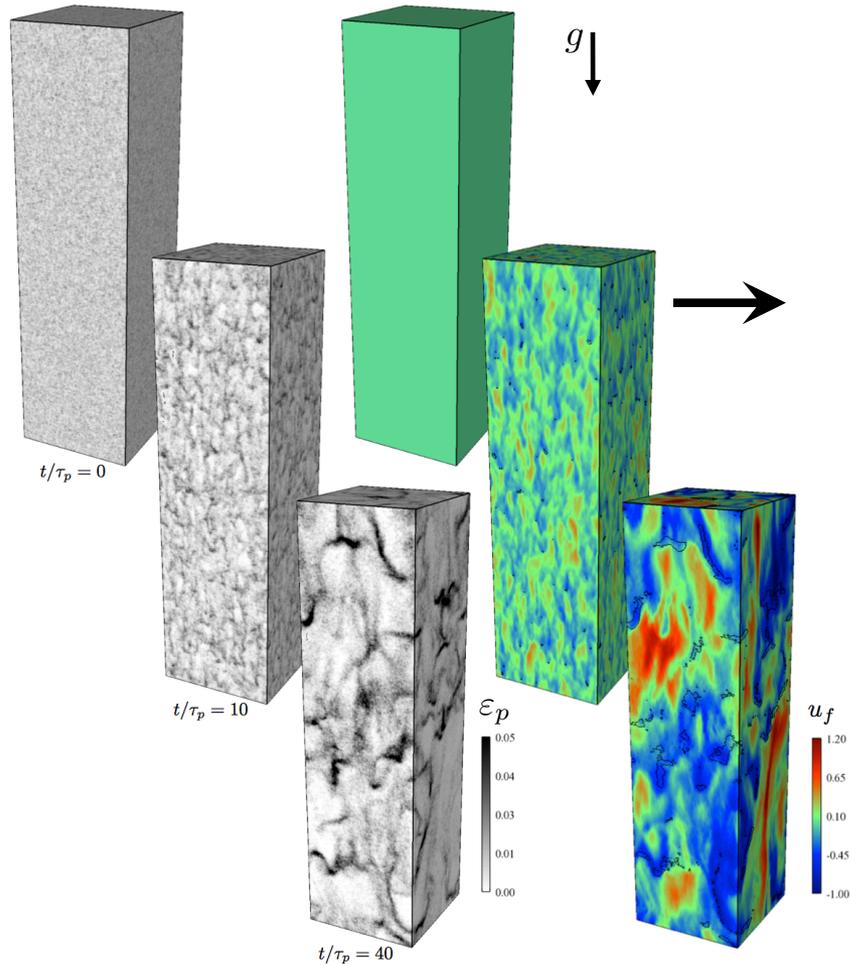
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# Fully-developed cluster-induced turbulence<sup>1</sup>



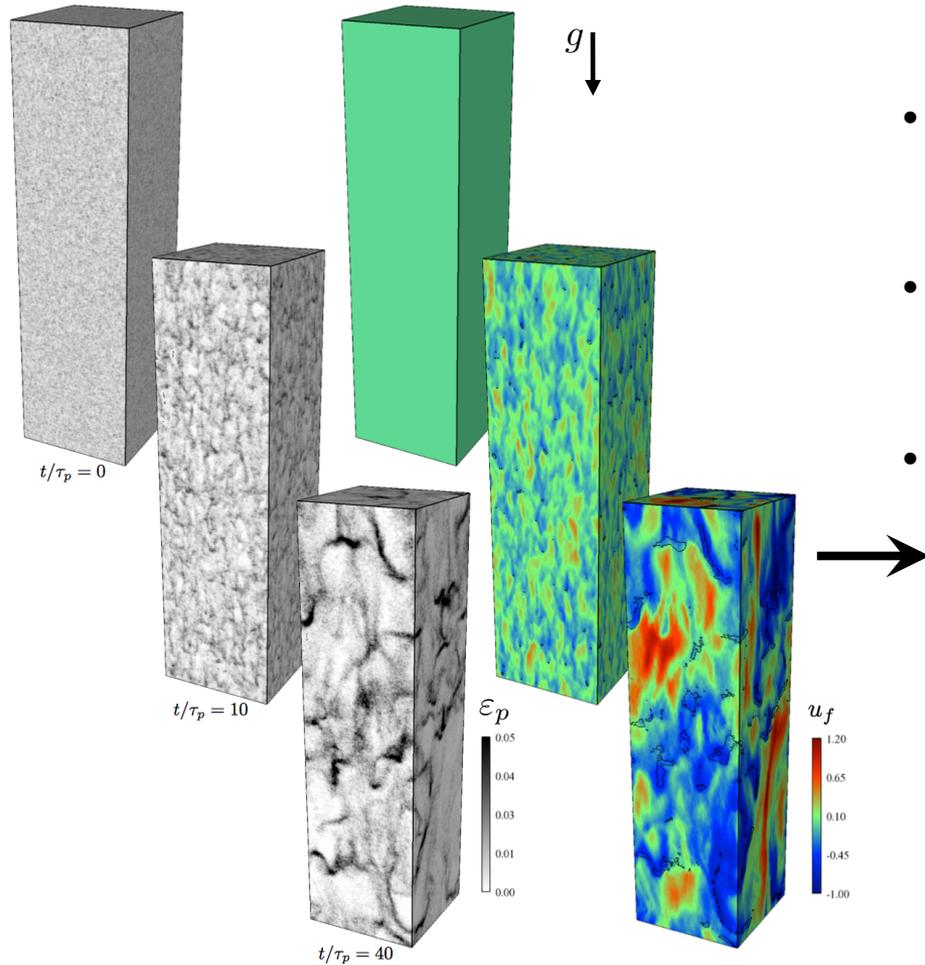
- Particles are initially randomly-distributed in a quiescent flow

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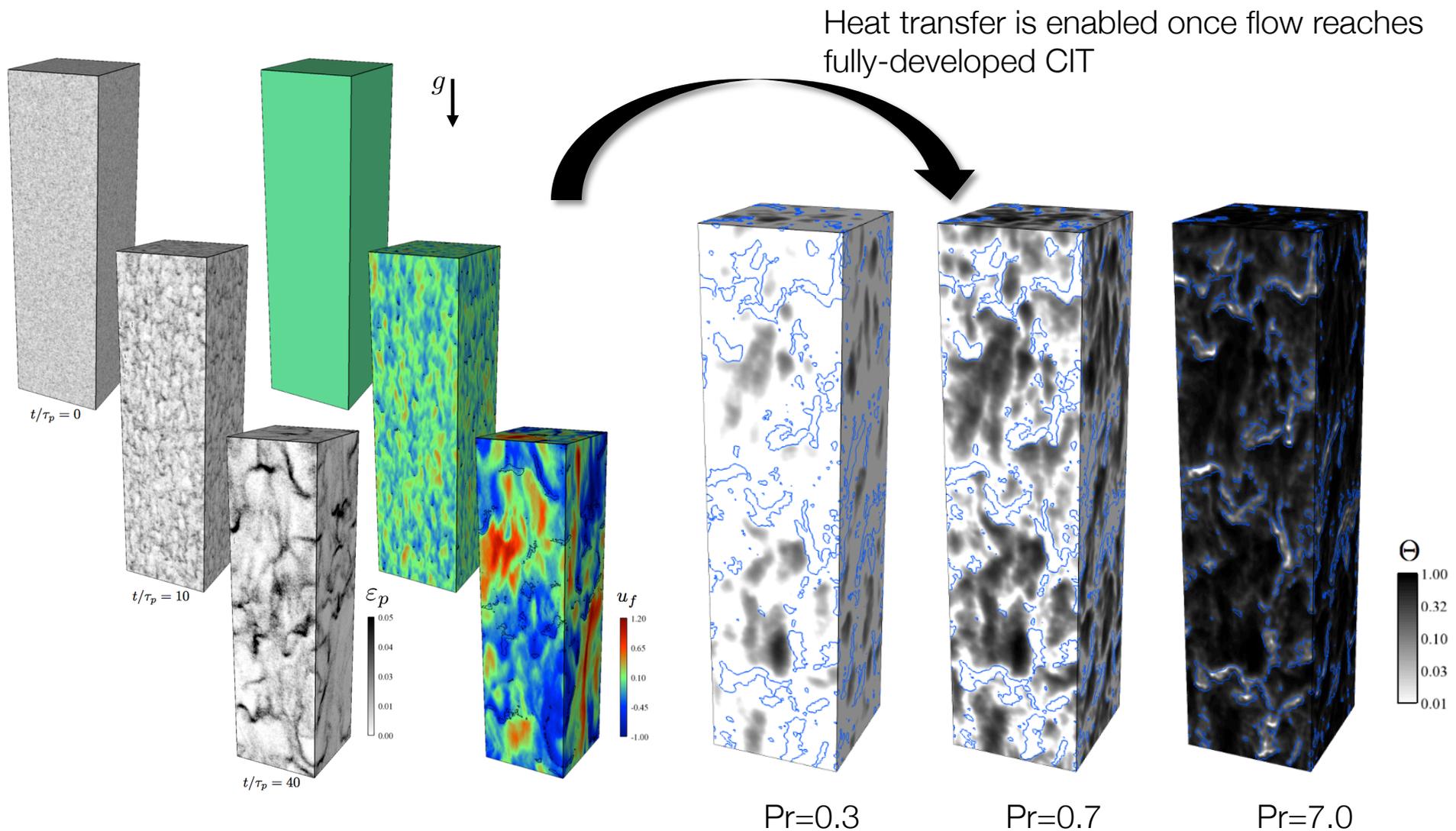
- Particles are initially randomly-distributed in a quiescent flow
- Particles settle under gravity. Two-way coupling spontaneously generate clusters

# Fully-developed cluster-induced turbulence<sup>1</sup>



- Particles are initially randomly-distributed in a quiescent flow
- Particles settle under gravity. Two-way coupling spontaneously generate clusters
- Fluctuations in volume fraction generate and sustain fluid-phase turbulence (fully-developed CIT)

# Fully-developed cluster-induced turbulence<sup>1</sup>



1. J. Capecelatro *et al.* (2014), *J. Fluid Mech.*

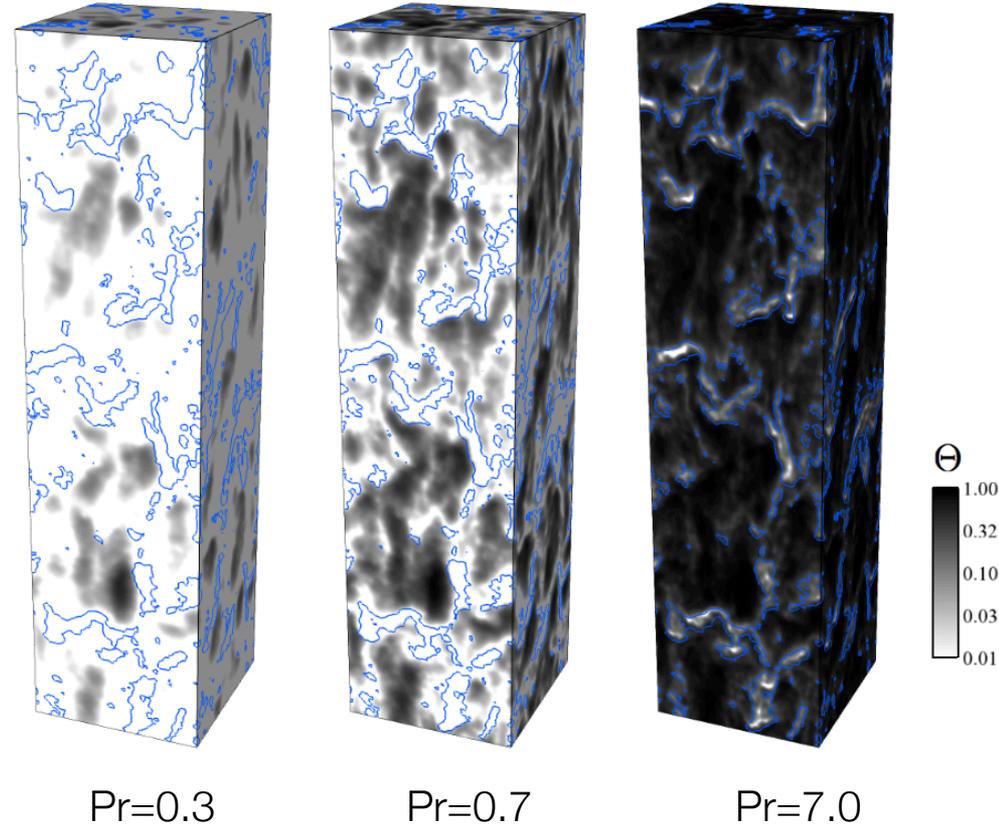
# Heat transfer in cluster-induced turbulence

- Initial fluid temperature:  $T_0 = 500$  K
- Initial particle temperature:  $T_p = 300$  K
- Non-dimensional temperature:

$$\Theta = \frac{T_f - T_p}{T_0 - T_p}$$

- Prandtl number:  $Pr = \frac{\mu}{\alpha_f \rho} f$
- Biot number:  $Bi = d_p h / k_p \ll 1$
- The gas phase cools relatively fast within clusters, while ‘hot spots’ persist in regions void of particles

Fluid temperature at  $t/\tau_p = 0.3$



# Heat transfer in cluster-induced turbulence

- Heat transfer is enhanced
- becomes fully developed

- Initial fluid temperature

- Initial particle temperature

- Non-dimensional

$$\theta = \frac{T_p - T_\infty}{T_0 - T_\infty}$$

- Prandtl number

- The gas phase is turbulent, while the motion of particles is laminar

$\langle \varepsilon_p \rangle$	$\phi$	$N_p$	Pr
$1 \times 10^{-3}$	1	85863	0.3
			0.7
			3.0
			7.0
$1 \times 10^{-2}$	10	858629	0.3
			0.7
			3.0
			7.0
$5 \times 10^{-2}$	50	4293144	0.3
			0.7
			3.0
			7.0

For all cases:  $\rho_p/\rho_f = 1000$ ,  $Re_p = 0.5$ , corresponding to 90  $\mu\text{m}$  particles in air.

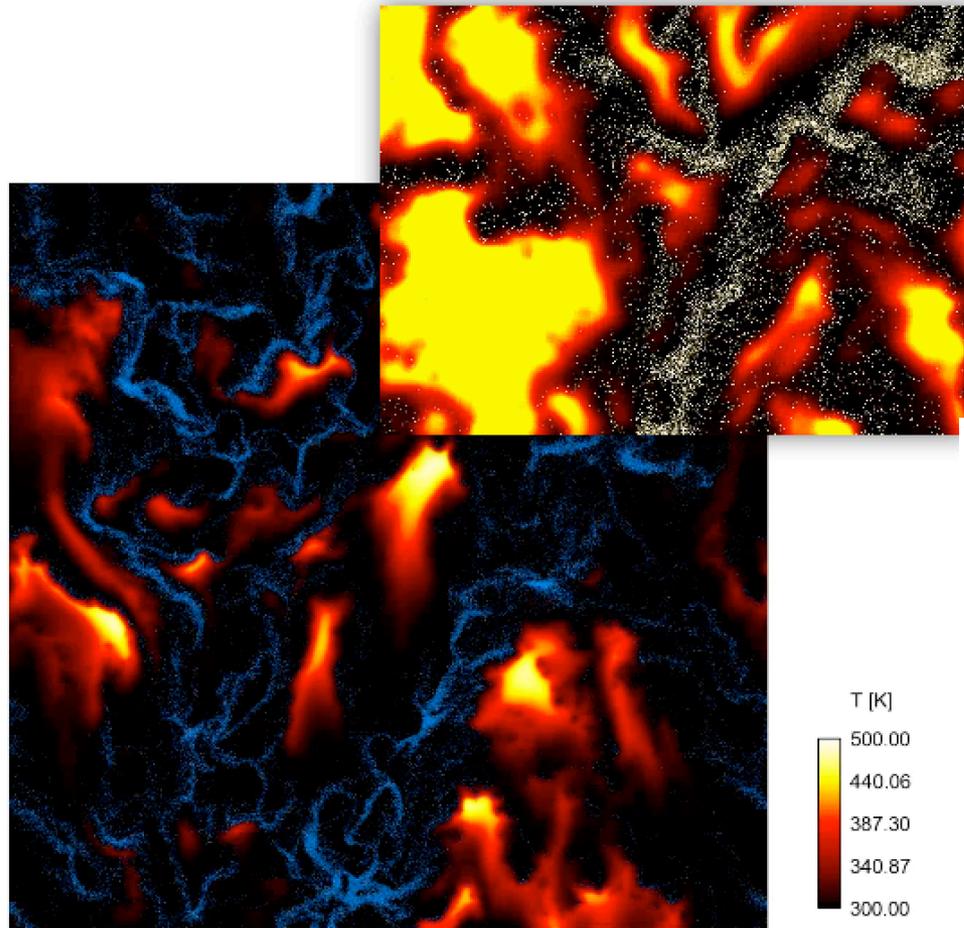
Mesh:  $512 \times 128 \times 128$ ,  $\Delta x = 1.75d_p$



0.0  
0.5  
1.0  
1.5  
2.0

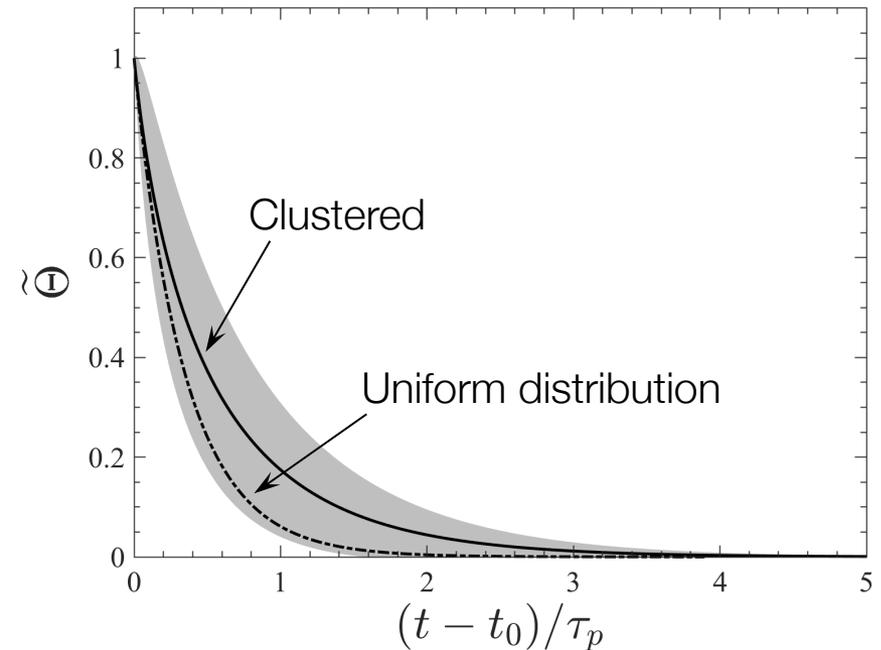


# Reduced thermal transport in gas-solid flows

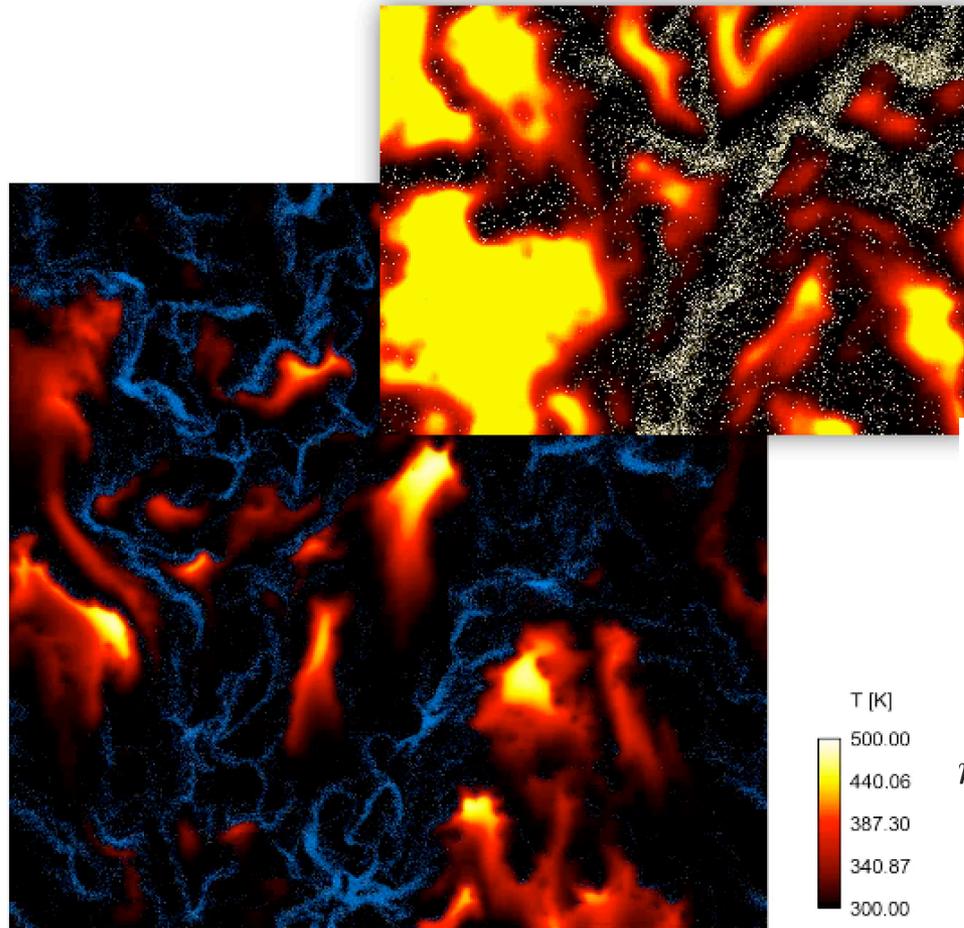


- Compared to a uniform particle distribution, clusters act to delay heat transfer between the two phases
- Non-dimensional temperature:

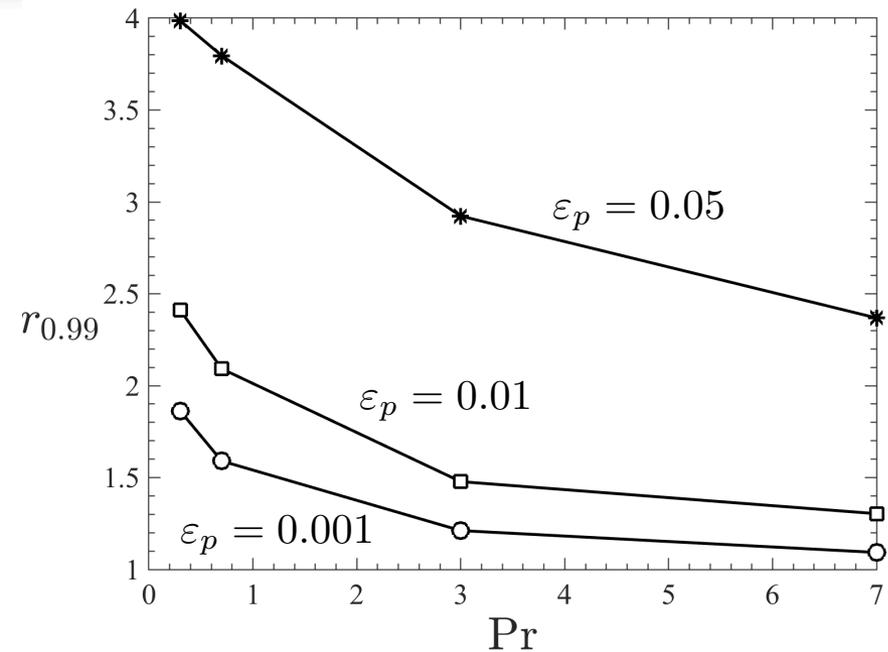
$$\Theta = \frac{T_f - T_p}{T_0 - T_p} \quad \tilde{\Theta} = \frac{\langle \varepsilon_f \Theta \rangle}{\langle \varepsilon_f \rangle}$$



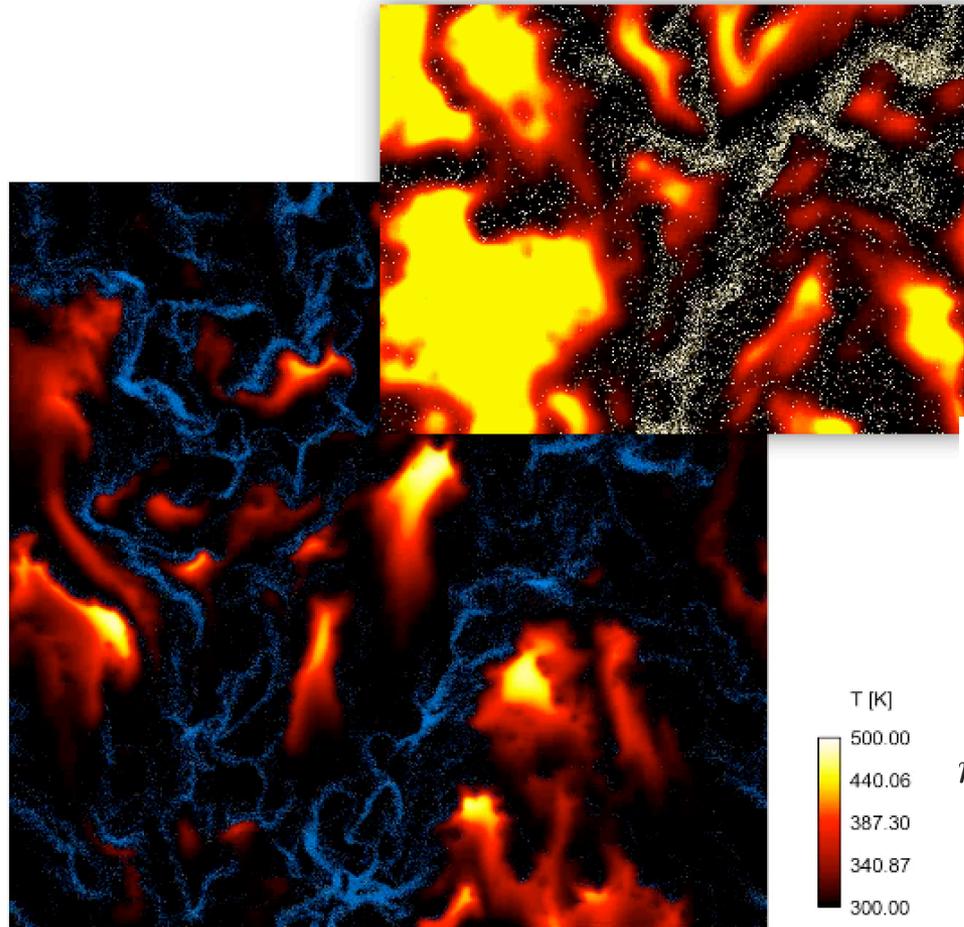
# Reduced thermal transport in gas-solid flows



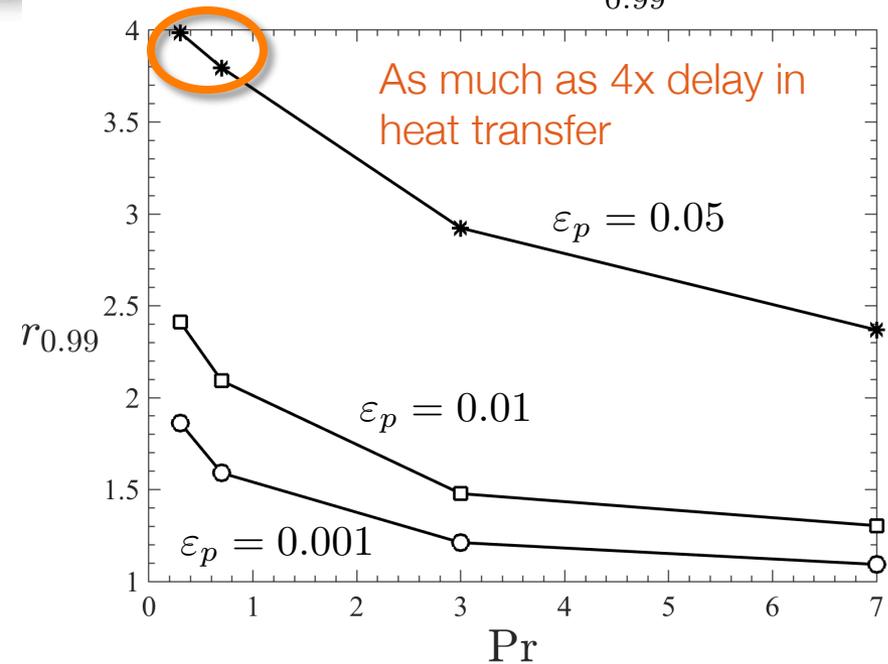
- Compared to a uniform particle distribution, clusters act to delay heat transfer between the two phases
- Reduction in heat transfer:
  - $\tau_{0.99}$  : Time to reach 99% ( $\tilde{\Theta} = 0.01$ )
  - Reduction:  $r_{0.99} = \frac{\tau_{0.99}^c}{\tau_{0.99}^h}$



# Reduced thermal transport in gas-solid flows



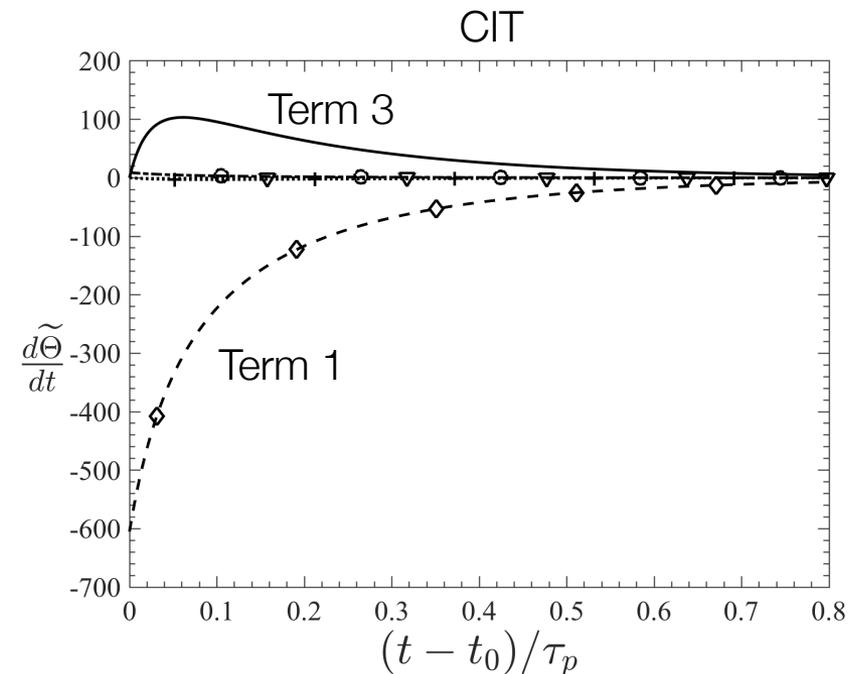
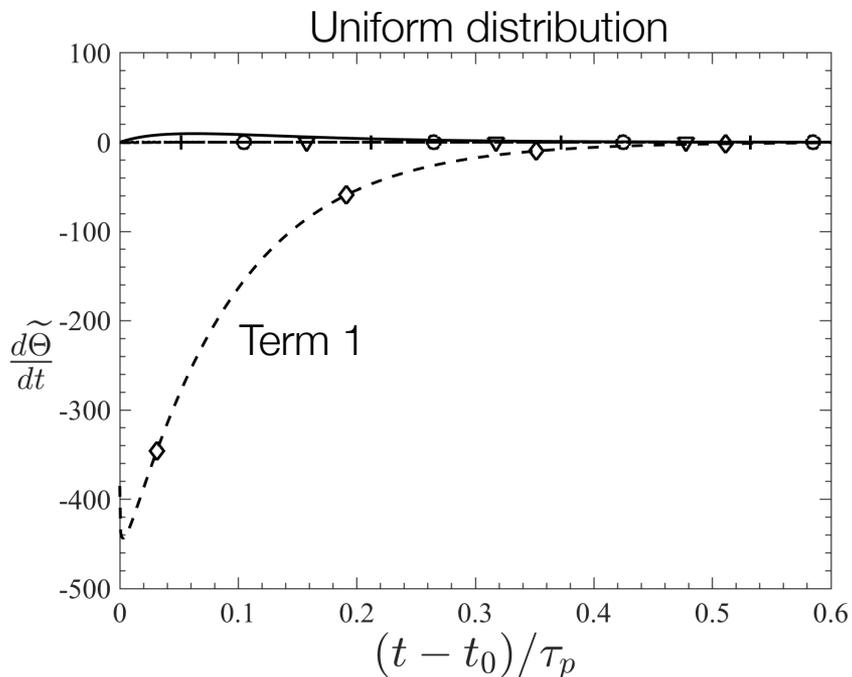
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# Reduced thermal transport in gas-solid flows

Evolution of the averaged non-dimensional temperature:

$$\frac{d\tilde{\Theta}}{dt} = \underbrace{-\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\varepsilon_p\rangle \langle\text{Nu}\rangle \langle\Theta\rangle}_{\text{Term 1}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\varepsilon_p\rangle \langle\text{Nu}'\Theta'\rangle}_{\text{Term 2}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\text{Nu}\rangle \langle\varepsilon'_p \Theta'\rangle}_{\text{Term 3}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\text{Nu}'\varepsilon'_p\rangle \langle\Theta\rangle}_{\text{Term 4}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\varepsilon'_p \text{Nu}'\Theta'\rangle}_{\text{Term 5}}$$

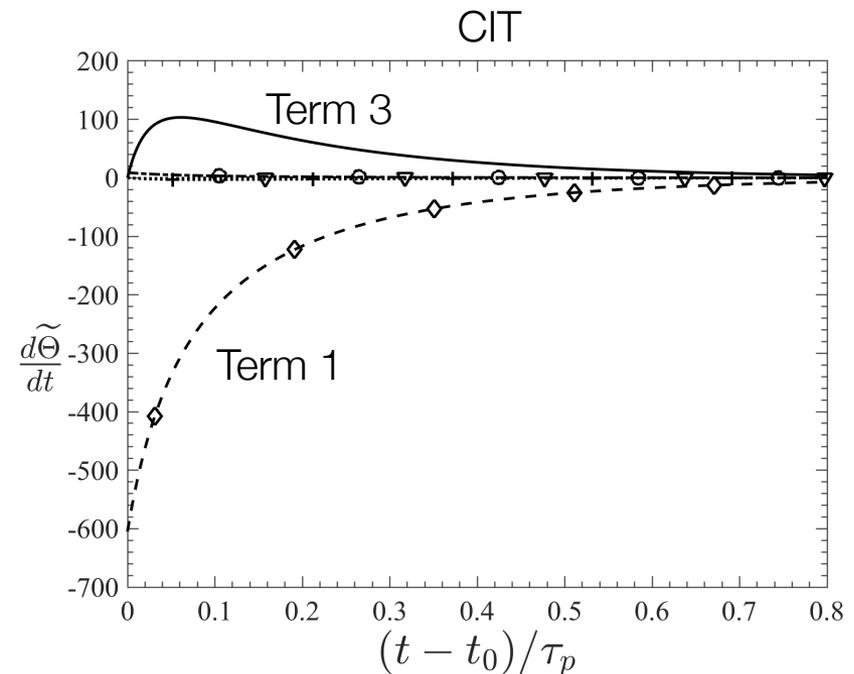
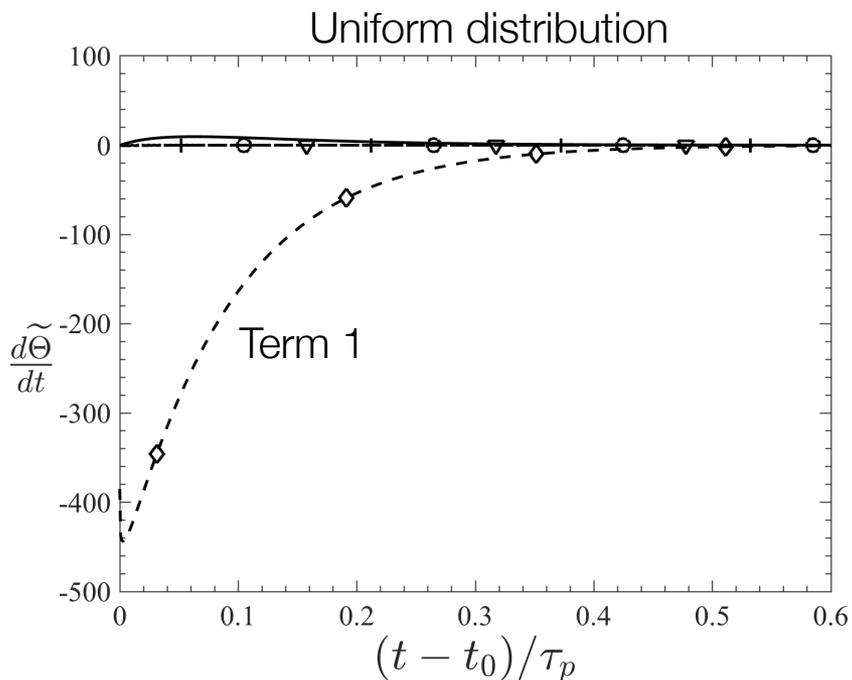


# Reduced thermal transport in gas-solid flows

Evolution of the averaged non-dimensional temperature:

$$\frac{d\tilde{\Theta}}{dt} = \underbrace{-\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\varepsilon_p\rangle \langle\text{Nu}\rangle \langle\Theta\rangle}_{\text{Term 1}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\varepsilon_p\rangle \langle\text{Nu}'\Theta'\rangle}_{\text{Term 2}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\text{Nu}\rangle \langle\varepsilon'_p \Theta'\rangle}_{\text{Term 3}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\text{Nu}'\varepsilon'_p\rangle \langle\Theta\rangle}_{\text{Term 4}} - \underbrace{\frac{6\alpha_f}{\langle\varepsilon_f\rangle d_p^2} \langle\varepsilon'_p \text{Nu}'\Theta'\rangle}_{\text{Term 5}}$$

Main term responsible for observed reduction



# Correcting for clusters using presume-shape PDFs

$$\langle \varepsilon_f \rangle \frac{\partial \tilde{\Theta}}{\partial t} = \frac{6\alpha_f}{d_p^2} \{ \langle \varepsilon_p \rangle \langle \text{Nu} \rangle \langle \Theta \rangle - \langle \varepsilon_p \rangle \langle \text{Nu}' \Theta' \rangle - \langle \text{Nu} \rangle \langle \varepsilon_p' \Theta' \rangle + \langle \text{Nu}' \varepsilon_p' \rangle \langle \Theta \rangle - \langle \varepsilon_p' \text{Nu}' \Theta' \rangle \}$$


  
 → Fluid temperature *seen by the particles*  
 → Primary source of suppressing heat transfer

- Can be obtained by integrating the joint-PDF  $f(\varepsilon_f, \Theta)$

$$\langle \varepsilon_p' \Theta' \rangle = \iint (\varepsilon_p - \langle \varepsilon_p \rangle) (\Theta - \langle \Theta \rangle) f(\varepsilon_f, \Theta) d\varepsilon_p d\Theta$$

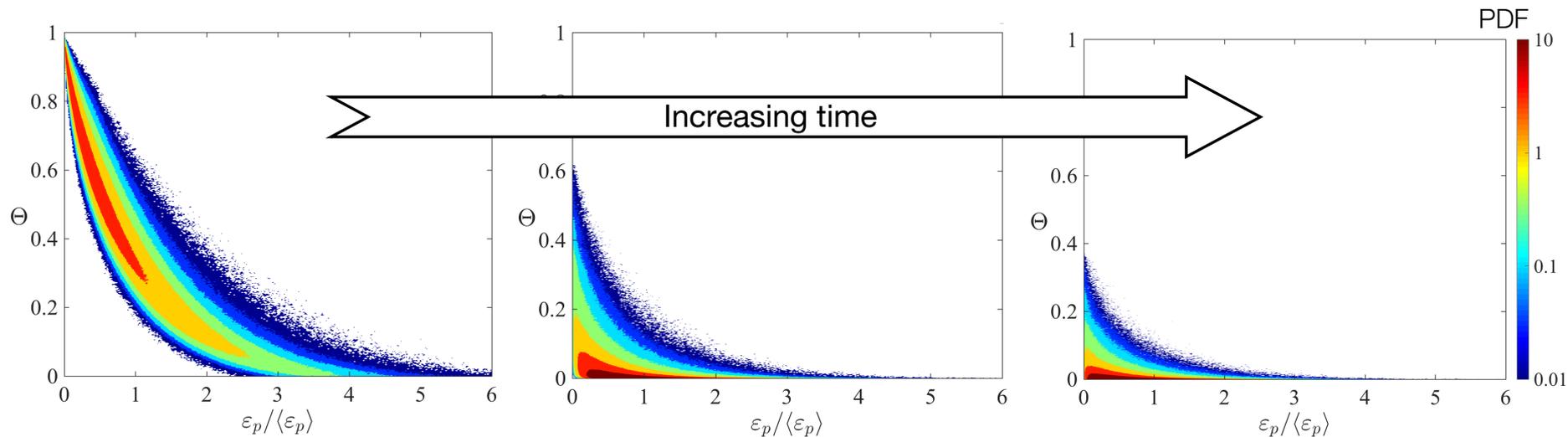
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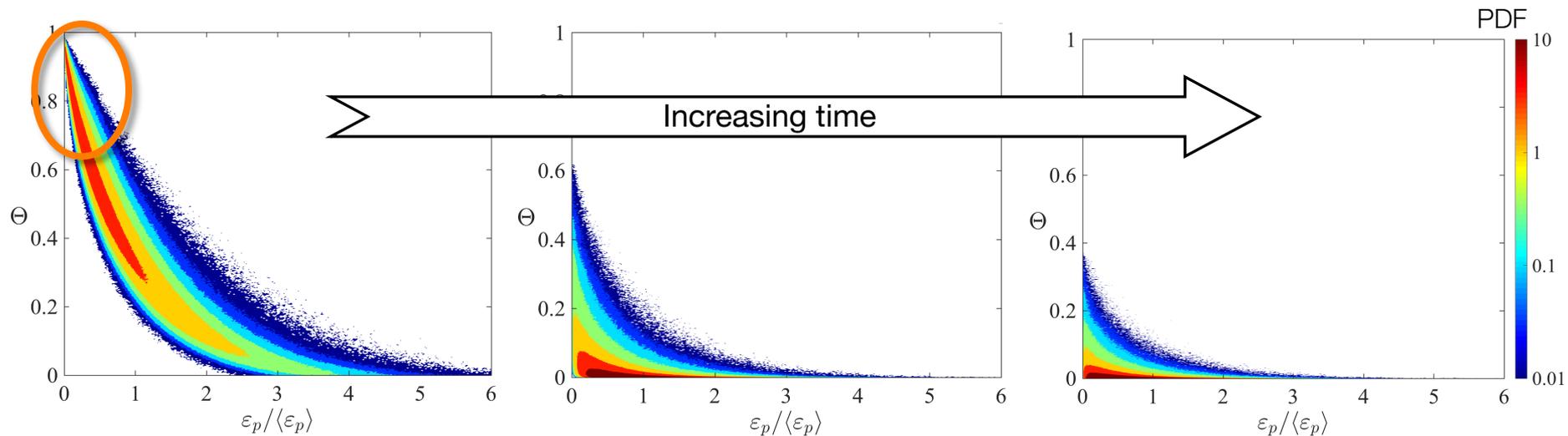
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At early times, high probability of high temperature in dilute regions

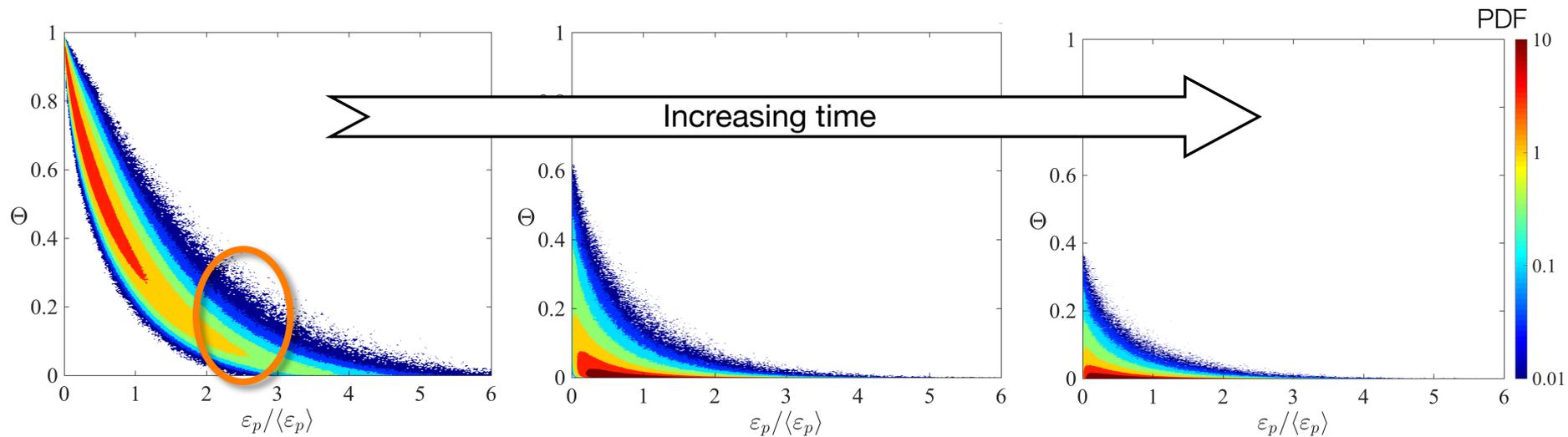
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- Can be obtained by integrating the joint-PDF  $f(\varepsilon_f, \Theta)$

$$\langle \varepsilon_p' \Theta' \rangle = \iint (\varepsilon_p - \langle \varepsilon_p \rangle)(\Theta - \langle \Theta \rangle) f(\varepsilon_f, \Theta) d\varepsilon_p d\Theta$$



At early times, low temperature only exists within clusters

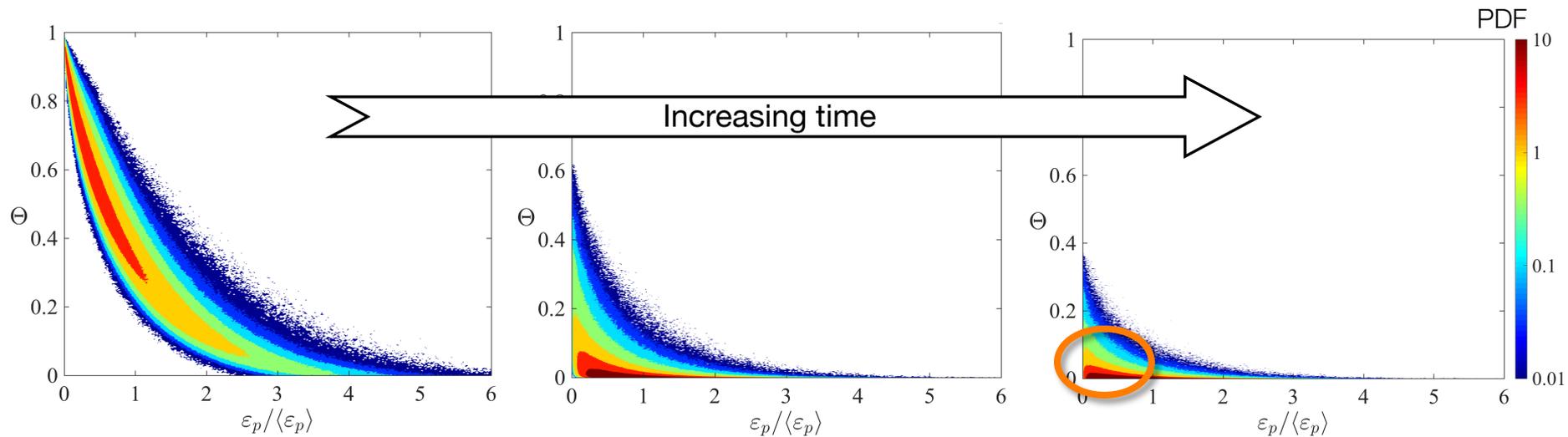
# Correcting for clusters using presume-shape PDFs

$$\langle \varepsilon_f \rangle \frac{\partial \tilde{\Theta}}{\partial t} = \frac{6\alpha_f}{d_p^2} \left\{ \langle \varepsilon_p \rangle \langle \text{Nu} \rangle \langle \Theta \rangle - \langle \varepsilon_p \rangle \langle \text{Nu}' \Theta' \rangle - \langle \text{Nu} \rangle \langle \varepsilon_p' \Theta' \rangle + \langle \text{Nu}' \varepsilon_p' \rangle \langle \Theta \rangle - \langle \varepsilon_p' \text{Nu}' \Theta' \rangle \right\}$$

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At late times, temperature is cooled everywhere

# Correcting for clusters using presume-shape PDFs

$$\langle \varepsilon_f \rangle \frac{\partial \tilde{\Theta}}{\partial t} = \frac{6\alpha_f}{d_p^2} \{ \langle \varepsilon_p \rangle \langle \text{Nu} \rangle \langle \Theta \rangle - \langle \varepsilon_p \rangle \langle \text{Nu}' \Theta' \rangle - \langle \text{Nu} \rangle \langle \varepsilon_p' \Theta' \rangle + \langle \text{Nu}' \varepsilon_p' \rangle \langle \Theta \rangle - \langle \varepsilon_p' \text{Nu}' \Theta' \rangle \}$$


  
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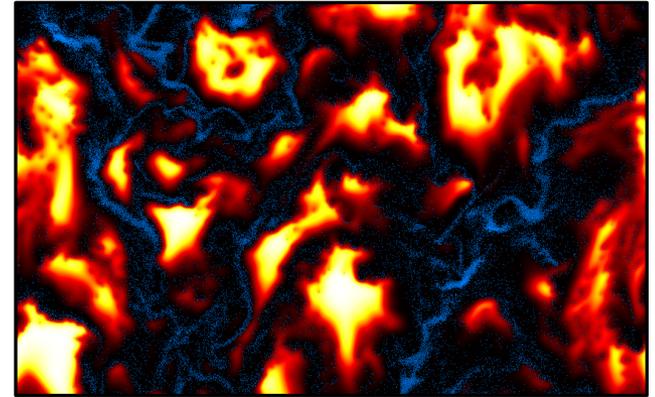
- Joint-PDF needs to be known for each  $\varepsilon_f, \Theta$  instance at all times!
  - Rewrite using Bayes' theorem:

$$f(\varepsilon_f, \Theta) = f(\Theta | \varepsilon_p) g(\varepsilon_p)$$

- We need a model for the volume fraction PDF  $g$  and conditional PDF  $f(\Theta | \varepsilon_p)$
- Preliminary work shows this can be accomplished using log-normal and beta distributions

# Towards a statistical description of thermal transport

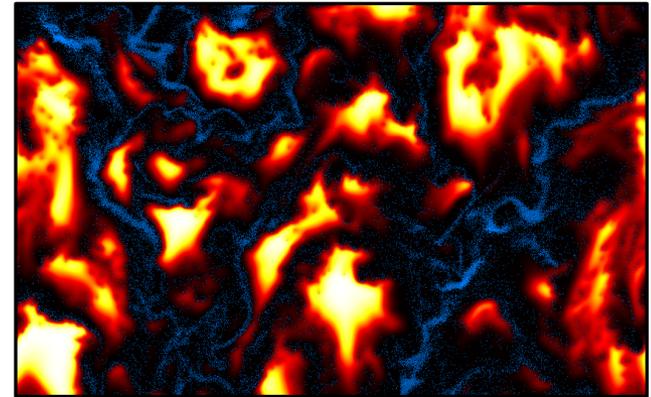
- To isolate the effects of interphase heat transfer in unsteady gas-solid flows, we consider fully-developed CIT of cold particles suspended in a hot gas
- Clusters were observed to delay heat transfer rate by as much as a factor of 4 (larger effect at higher mean concentration and lower Pr).



# Towards a statistical description of thermal transport

- To isolate the effects of interphase heat transfer in unsteady gas-solid flows, we consider fully-developed CIT of cold particles suspended in a hot gas
- Clusters were observed to delay heat transfer rate by as much as a factor of 4 (larger effect at higher mean concentration and lower Pr).
- A mean temperature difference can be enforced by adding a heat source that uniformly replenishes heat lost in the gas phase (thermally fully-developed CIT)

$$\dot{Q}(t) = -\frac{1}{V} \int_V \mathcal{Q}_{\text{inter}}(\mathbf{x}, t) dV$$

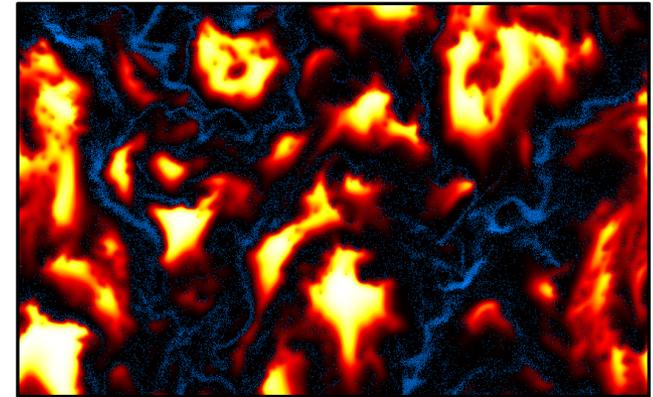


# Towards a statistical description of thermal transport

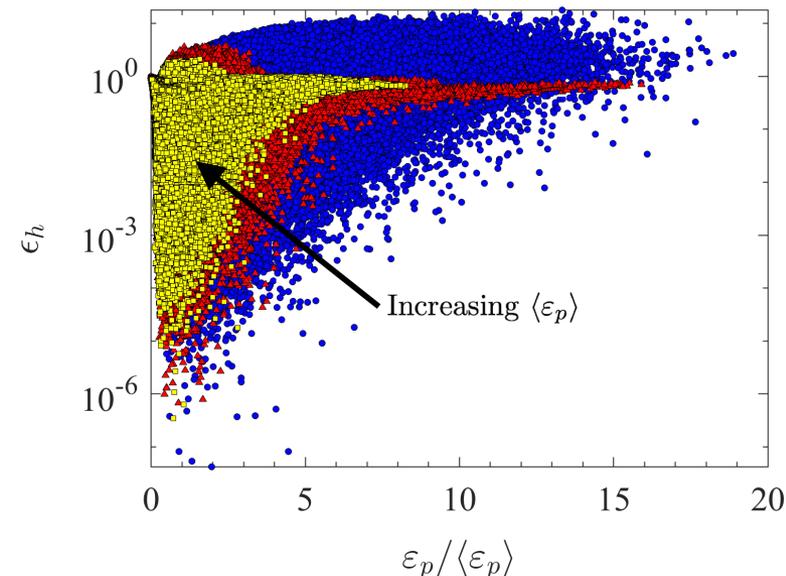
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$$\dot{Q}(t) = -\frac{1}{V} \int_V \mathcal{Q}_{\text{inter}}(\mathbf{x}, t) dV$$

- Local heat transfer coefficient is seen to vary by many orders of magnitude due to clusters

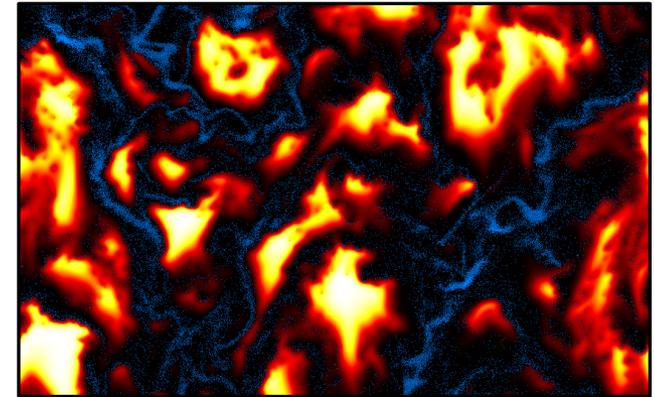


$$\epsilon_h = \left| 1 - \frac{\langle \epsilon_p \text{Nu} (T_f - T_p) \rangle}{\langle \epsilon_p \rangle \langle \text{Nu} \rangle (\langle T_f \rangle - \langle T_p \rangle)} \right|$$



# Towards a statistical description of thermal transport

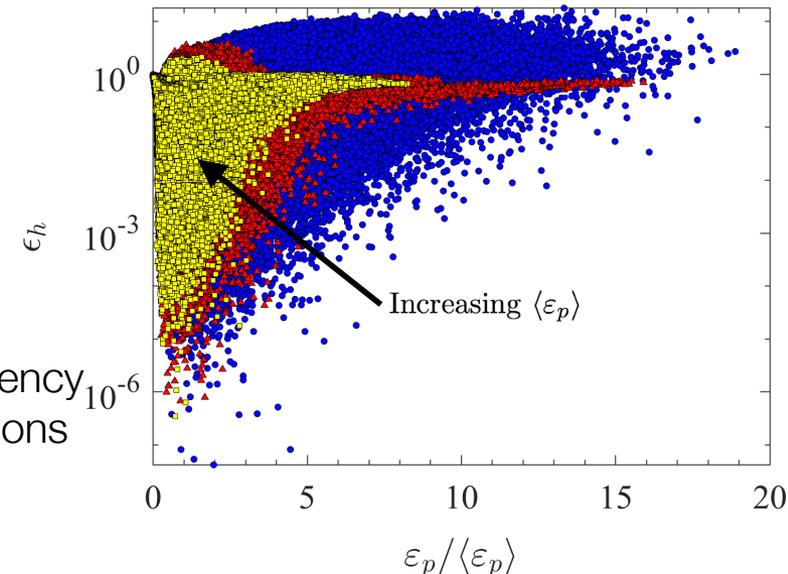
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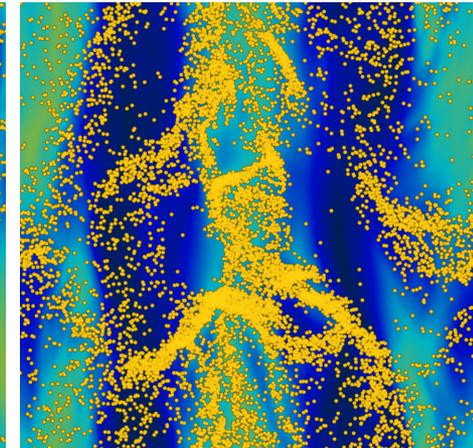
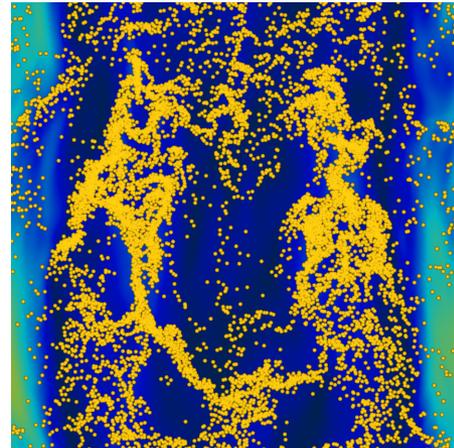
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$$\dot{Q}(t) = -\frac{1}{V} \int_V \mathcal{Q}_{\text{inter}}(\mathbf{x}, t) dV$$

- Local heat transfer coefficient is seen to vary by many orders of magnitude due to clusters
- Volume fraction-temperature covariance  $\langle \epsilon'_p \Theta' \rangle$  is the primary term responsible for reduced heat transfer efficiency
  - Presumes PDF using log-normal and beta distributions are being tested to model this term statistically



# Questions?



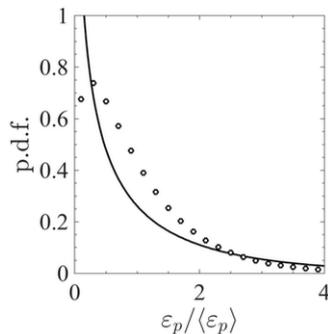
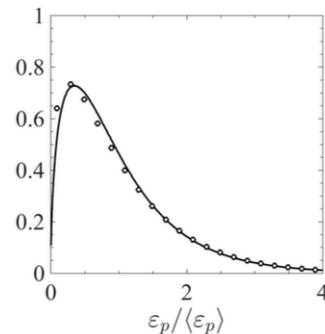
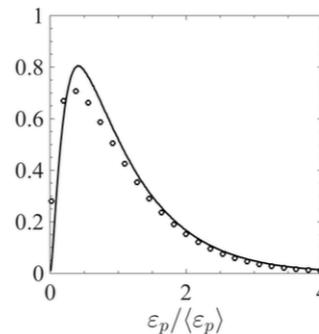
# Backup slides

# Thermally fully-developed CIT

- In order to investigate the steady-state behavior, add heat source to maintain constant mean temperature difference

$$\dot{Q}(t) = -\frac{1}{V} \int_V \mathcal{Q}_{\text{inter}}(\mathbf{x}, t) dV$$

- Volume fraction PDF  $g$  modeled using a beta distribution

(a)  $\langle \varepsilon_p \rangle = 0.001$ (b)  $\langle \varepsilon_p \rangle = 0.01$ (c)  $\langle \varepsilon_p \rangle = 0.05$ 

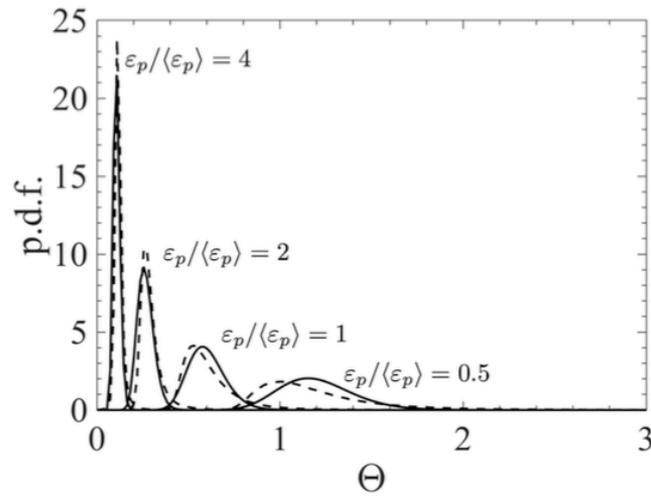
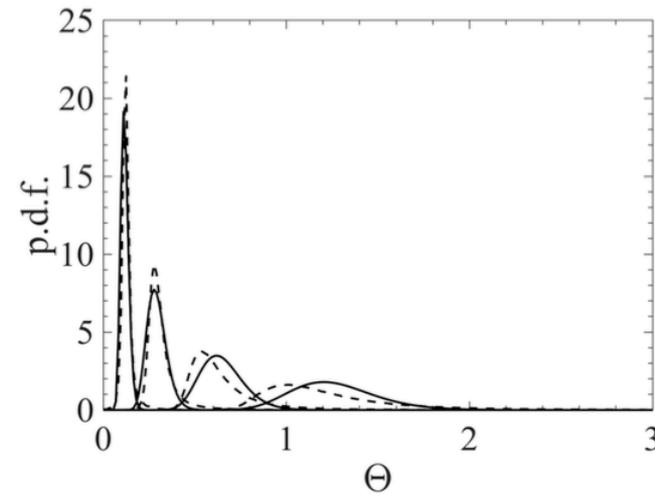
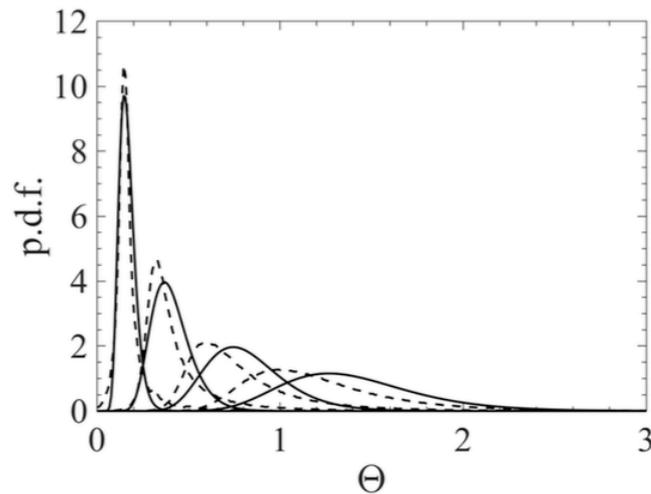
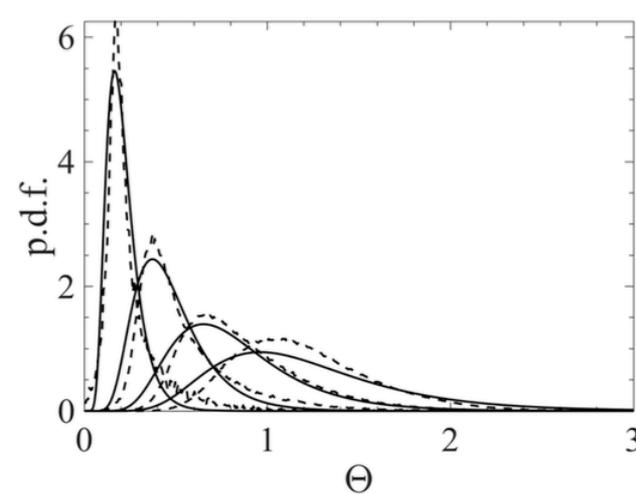
$$g(\langle \varepsilon_p \rangle) = \frac{\langle \varepsilon_p \rangle^{a-1} (1 - \langle \varepsilon_p \rangle)^{b-1}}{B(a, b)}$$

$$B(a, b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx$$

- The conditional PDF no longer varies with time, can be modeled using a log-normal distribution

$$f(\Theta | \varepsilon_p) = \frac{1}{\Theta \sigma_{\ln} \sqrt{2\pi}} \exp\left(-\frac{(\ln \Theta - \mu_{\ln})^2}{2\sigma_{\ln}^2}\right)$$

# Conditional PDF modeled using a log-normal distribution

(a)  $\text{Pr} = 0.3$ (b)  $\text{Pr} = 0.7$ (c)  $\text{Pr} = 3$ (d)  $\text{Pr} = 7$

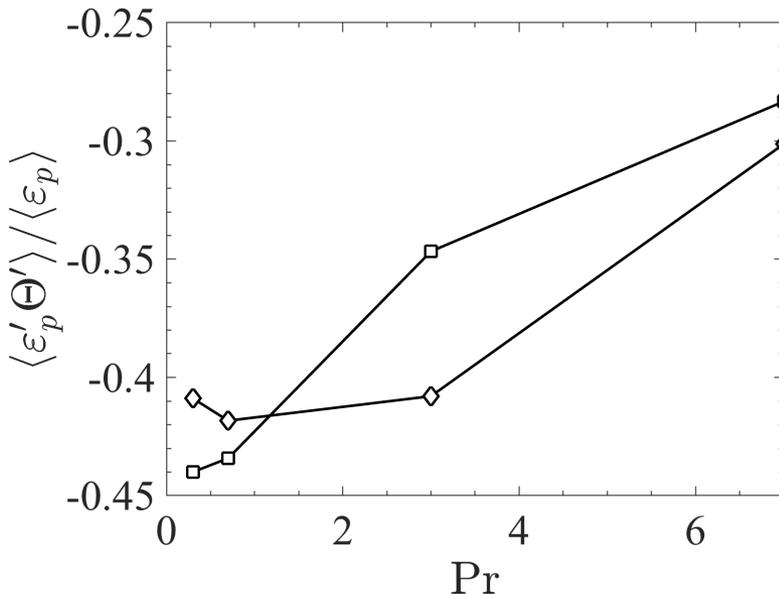
# Putting it all together

$$\langle \varepsilon_f \rangle \frac{\partial \tilde{\Theta}}{\partial t} = \frac{6\alpha_f}{d_p^2} \left\{ \langle \varepsilon_p \rangle \langle \text{Nu} \rangle \langle \Theta \rangle - \langle \varepsilon_p \rangle \langle \text{Nu}' \Theta' \rangle - \langle \text{Nu} \rangle \langle \varepsilon_p' \Theta' \rangle + \langle \text{Nu}' \varepsilon_p' \rangle \langle \Theta \rangle - \langle \varepsilon_p' \text{Nu}' \Theta' \rangle \right\}$$

→ Fluid temperature *seen by the particles*  
 → Primary source of augmenting heat transfer

- Can be obtained by integrating the joint-PDF  $f(\varepsilon_f, \Theta)$

$$\langle \varepsilon_p' \Theta' \rangle = \iint (\varepsilon_p - \langle \varepsilon_p \rangle)(\Theta - \langle \Theta \rangle) f(\varepsilon_f, \Theta) d\varepsilon_p d\Theta$$



- Rewrite using Bayes' theorem:

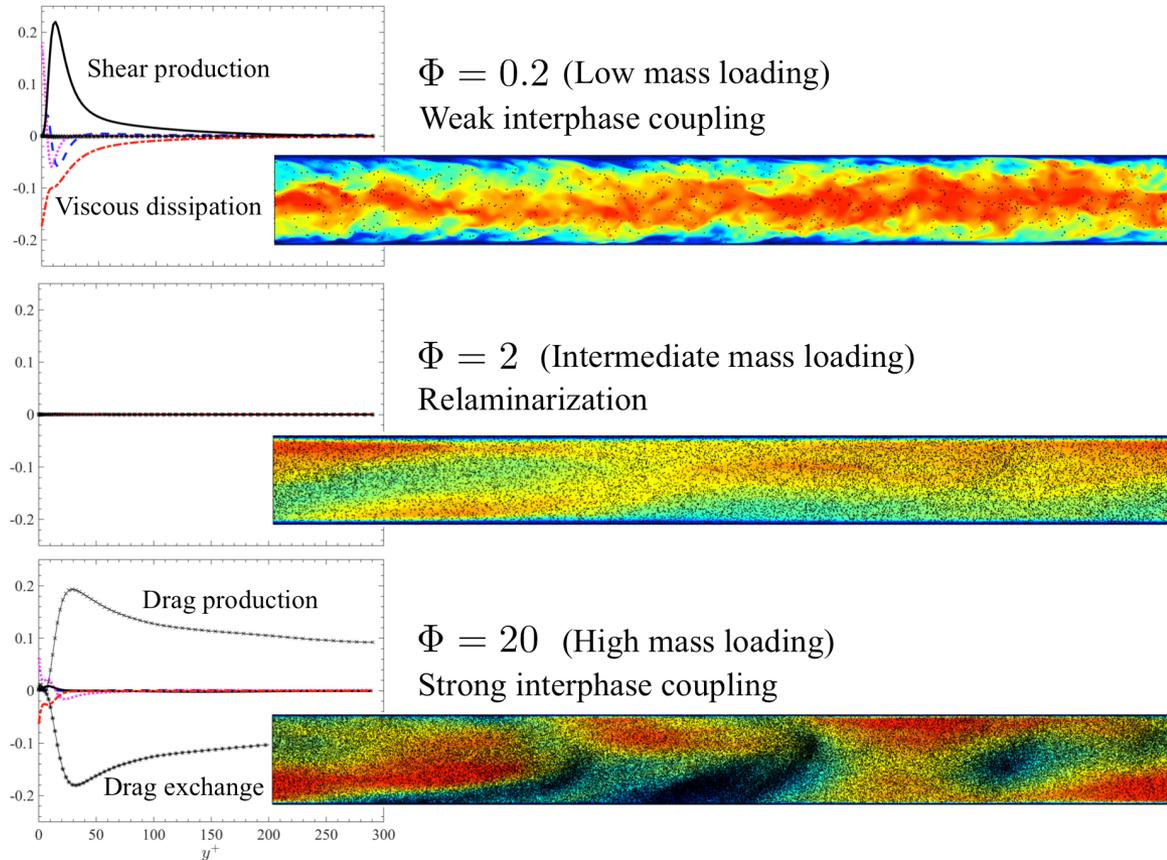
$$f(\varepsilon_f, \Theta) = f(\Theta | \varepsilon_p) g(\varepsilon_p)$$

Log-normal distribution

Beta distribution

# How does clustering influence heat transfer?

For a fixed Reynolds number in a particle-laden channel flow, increasing mass loading results in vastly different flow regimes that greatly impact fluid turbulent kinetic energy.<sup>1</sup>



*Focus of this talk: what is the role of multiphase interactions on heat transfer?*

1. J. Capecelatro, O. Desjardins, R.O. Fox (2018) *Journal of Fluid Mechanics*.